MATH 152
Summer 2016

SAMPLE EXAM II

1. Find the coefficient of $(x-4)^3$ in the Taylor Series expansion for the function $f(x) = \sqrt{x}$ at $a = 4$.

   a) $\dfrac{3}{256}$

   b) $\dfrac{3}{128}$

   c) $\dfrac{1}{256}$

   d) $\dfrac{5}{264}$

   e) $\dfrac{1}{512}$

2. Suppose we use s_4, the fourth partial sum, to approximate the sum of the series $S = \sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^4}$. Which of the following is true from the Alternating Series Estimation Theorem regarding an upper bound of the absolute value of the remainder?

   a) $|R_4| < \dfrac{1}{625}$

   b) $|R_4| < \dfrac{1}{256}$

   c) $|R_4| < \dfrac{1}{192}$

   d) $|R_4| < \dfrac{1}{375}$

   e) $|R_4| < \dfrac{1}{132}$

3. If $\sum_{n=1}^{\infty} c_n x^n$ converges at $x = 2$ and diverges at $x = 8$, then which of the following is true? Circle all true answers.

   a) $\sum_{n=1}^{\infty} c_n x^n$ will converge at $x = 1$

   b) $\sum_{n=1}^{\infty} c_n x^n$ will converge at $x = 3$

   c) $\sum_{n=1}^{\infty} c_n x^n$ will diverge at $x = 4$

   d) $\sum_{n=1}^{\infty} c_n x^n$ will diverge at $x = 9$

   e) $\sum_{n=1}^{\infty} c_n x^n$ may converge at $x = 6$.

4. The series $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ is

a) divergent because $\frac{1}{n-\sqrt{n}} > \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{n}$ is a divergent p-series.

b) convergent because $\frac{1}{n-\sqrt{n}} < \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{n}$ is a convergent p-series.

c) convergent because $\frac{1}{n-\sqrt{n}} > \frac{1}{n}$ and $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ is a convergent p series.

d) divergent by the Ratio Test.

e) divergent by the Test for Divergence.

5. Which of the following series converges?

(I) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ (II) $\sum_{n=1}^{\infty} \frac{n!}{n^{1000}}$ (III) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln\, n)^2}$

a) I only
b) II only
c) III only
d) I and III
e) all 3 series converge

6. Which of the following series is absolutely convergent?

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

c) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln\, n}$

d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

e) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^{10}+n}$

7. Find the radius of convergence of the power series $\sum_{n=0}^{\infty}(2n+1)!(2x-1)^n$

a) 0

b) ∞

c) $\frac{1}{2}$

d) $-\frac{1}{2}$

e) 1

8. Find the interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{(2n)!(2x-1)^n}{n!}$

a) $\left\{\frac{1}{2}\right\}$

b) $(-\infty,\infty)$

c) $\{0\}$

d) $(-1,1)$

e) Diverges everywhere

9. Which of the following is a Maclaurin Series for $\int_0^x \sin(t^2)\,dt$?

a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n}}{(2n)!}$

b) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$

c) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+4}}{(2n+4)(2n+1)!}$

d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!}$

e) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+3}}{(2n+3)(2n)!}$

10. Find the limit of the sequence $a_n = (-1)^n e^{\frac{\ln n}{n}}$

a) $\frac{1}{e}$

b) diverges

c) 0

d) 1

e) -1

11. $\sum_{n=1}^{\infty} \frac{3^{2n}}{(-10)^n} =$

a) $\frac{10}{19}$

b) $\frac{9}{19}$

c) $\frac{1}{8}$

d) -10

e) $\frac{-9}{19}$

12. The series $\sum_{n=1}^{\infty} \frac{n}{3n+1}$

a) Converges to $\frac{1}{3}$

b) Converges to 0

c) Converges to 1

d) Converges to $\frac{1}{4}$

e) diverges

13. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n+1}{n+2}$, find (i) a_n and (ii) $\sum_{n=1}^{\infty} a_n$.

a) (i) $a_n = \frac{1}{(n+1)(n+2)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

b) (i) $a_n = \frac{2}{(n+1)(n+2)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

c) (i) $a_n = \frac{1}{(n+1)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

d) (i) $a_n = \frac{n}{(n+1)(n+2)}$ and (ii) $\sum_{n=0}^{\infty} a_n = 1$

e) None of these, the series diverges by the Test for Divergence.

14. Find the length of the curve $y = 4x^{3/2}$ from $(0, 0)$ to $(2, 4)$.

a) $\frac{1}{54}(73\sqrt{73} - 1)$

b) $\frac{1}{27}(73\sqrt{73} - 1)$

c) $\frac{1}{54}(37\sqrt{37} - 1)$

d) $\frac{1}{27}(37\sqrt{37} - 1)$

e) None of these.

15. Consider the sequence whose terms are defined as $a_1 = 2$, $a_{n+1} = 4 - \frac{3}{a_n}$. Given that the terms of the sequence are increasing and bounded, find the limit of the sequence.

a) $L = 1$

b) $L = 3$

c) $L = 2.5$

d) $L = 2.8$

e) The sequence diverges.

Part II - Work Out Problems

16. Find the surface area obtained by rotating the curve parametrized by $x(t) = \cos^2 t,\ y(t) = \sin^2 t,\ 0 \le t \le \frac{\pi}{2}$ about the y axis.

17. Find the sum of the series

(i) The series whose nth partial sum is given by $s_n = \ln(n) - \ln(2n+1)$

(ii) $\sum_{n=0}^{\infty} \frac{2^{2n+3}}{5^n}$

(iii) $\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$

18. Find the surface area obtained by rotating the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$, $1 \leq x \leq 3$, about the x axis.

19. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n2^n}$

20. Find a power series for $g(x) = \int x^3 \ln(1 + x^2)\, dx$.

21. Find the power series about 0 for $f(x) = \dfrac{2x}{(1 - x^2)^2}$

22. Find the Taylor Series for $f(x) = \dfrac{1}{x+1}$ centered at $a = 1$.

23. Prove the series $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges. Use the sum of the first 4 terms to approximate the sum of the series and estimate the error.