Summer 2016 Math 152

Week in Review 3
courtesy: Amy Austin
(covering 9.3,9.4,10.1,10.2)

Section 9.3

1. Find the length of the curve $y = 2x^{3/2}$, $0 \le x \le \frac{1}{4}$.
2. Find the length of the curve $x = y^2 - \frac{\ln(y)}{8}$ from $y = 1$ to $y = e$.
3. Find the length of the parametric curve $x = 3t - t^3$, $y = 3t^2$, $0 \le t \le 2$.

Section 9.4

4. Find the surface area obtained by revolving the given curve about the indicated axis.
   a.) $y = 2x^3$, $0 \le x \le 1$ about the x axis.
   b.) $y^2 = x + 2$, $1 \le y \le 3$ about the x axis.
   c.) $y = x^2 + 1$, $0 \le x \le 1$, about the y axis.
   d.) $y = \sqrt{4x}$, $0 \le x \le 1$, about the x axis.
   e.) $x = \sin(3t)$, $y = \cos(3t)$, $0 \le t \le \frac{\pi}{12}$. about the y axis.

Section 10.1

5. Find the fourth term of the sequence $\{\frac{n}{n+1}\}_{n=2}^{\infty}$
6. Find a general formula for the sequence
   $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$
7. Determine whether the following sequences converge or diverge. If the sequence converges, find the limit. If the sequence diverges, explain why.
   a.) $a_n = \frac{n^3}{n^2 + 500n - 2}$
   b.) $a_n = \ln(2n + 1) - \ln(5n + 4)$
   c.) $a_n = \frac{(-1)^n}{n}$
   d.) $a_n = \frac{5 \cos n}{n}$
   e.) $a_n = \frac{(-1)^n n}{5n + 6}$
8. Prove the sequence $a_n = \frac{\ln n}{n}$ is a decreasing sequence.
9. For the recursive sequence given, find the 3rd term and find the value of the limit.
   $a_1 = 2$, $a_{n+1} = 2 + \frac{1}{4}a_n$.

Section 10.2

Definition: A series is the sum of a sequence, that is $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_{100} + \ldots + \ldots$.

Definition: We call $\{s_n\}$ a sequence of partial sums, where

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

Definition: $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_i$

10. Find the first 5 terms in the sequence of partial sums the series $\sum_{n=1}^{\infty}(1)$. From this, what can be concluded about the convergence/divergence of this series?
11. Find the first 5 terms in the sequence of partial sums the series $\sum_{n=1}^{\infty}(-1)^n$. From this, what can be concluded about the convergence/divergence of this series?
12. Use the Test For Divergence to show the series diverges: $\sum_{n=1}^{\infty} \frac{n^2}{3(n+1)(n+2)}$
13. Explain why the Test for Divergence is inconclusive when applied to the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$.
14. If the n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is
    $s_n = \frac{n+1}{n+4}$, find:
    a.) s_{100}, that is $\sum_{n=1}^{100} a_n = ?$
    b.) The sum of the series, that is $\sum_{n=1}^{\infty} a_n = ?$
    c.) A general formula for a_n, then find a_6.
15. Find the sum of the series:
    a.) $\sum_{n=1}^{\infty} \left(\sin\frac{1}{n} - \sin\frac{1}{n+1}\right)$
    b.) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$
    c.) $\sum_{n=1}^{\infty} 2\left(\frac{5}{7}\right)^{n-1}$
    d.) $\sum_{n=0}^{\infty} \frac{2^{3n}}{(-5)^{n+1}}$
    e.) $\sum_{n=0}^{\infty} \frac{(-1)^n + 3^n}{4^n}$