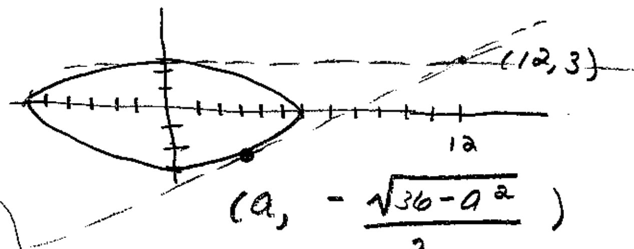
Section 3.6: Implicit Differentiation: (added problem to assist with webassign HW 14 problem 5)

Find the equations of both the tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$.

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$



one tangent line is $y = 3$.

now, $4y^2 = 36 - x^2$

$8y\,y' = -2x$

$$y' = \frac{-x}{4y}$$

$$m = \frac{-a}{4\frac{\sqrt{36-a^2}}{2}}$$

$4y^2 = 36 - x^2$

$$y^2 = 9 - \frac{x^2}{4} = \frac{36 - x^2}{4}$$

$$y = \frac{\sqrt{36 - x^2}}{2}$$

$$\boxed{m = \frac{3 + \frac{\sqrt{36-a^2}}{2}}{12 - a}}$$

equate

$$\boxed{m = \frac{a}{2\sqrt{36-a^2}}}$$

$$\frac{3 + \frac{\sqrt{36-a^2}}{2}}{12 - a} = \frac{a}{2\sqrt{36-a^2}}$$

$$\left(3 + \frac{\sqrt{36-a^2}}{2}\right)\left(2\sqrt{36-a^2}\right) = a(12 - a)$$

$$6\sqrt{36-a^2} + 36 - \cancel{a^2} = 12a - \cancel{a^2}$$

$$\sqrt{36-a^2} + 6 = 2a$$

$$\sqrt{36-a^2} = 2a - 6$$

$$\cancel{36} - a^2 = 4a^2 - 24a + \cancel{36}$$

$$0 = 5a^2 - 24a$$

$$= a(5a - 24)$$

$$a = 0, \quad a = \frac{24}{5}$$

$a = 0$ (already knew by horizontal tangent)

$$a = \frac{24}{5}, \quad y = \frac{\sqrt{36 - \left(\frac{24}{5}\right)^2}}{2}$$

$$y = \frac{18}{10} = \frac{9}{5}$$

point $= \left(\frac{24}{5}, \frac{9}{5}\right)$

$$m = \frac{a}{2\sqrt{36-a^2}} = \frac{24/5}{2\sqrt{36 - \left(\frac{24}{5}\right)^2}} = \frac{24/5}{2 \cdot \frac{18}{5}}$$

$$\boxed{m = \frac{2}{3}}$$

$$\boxed{y - \frac{9}{5} = \frac{2}{3}\left(x - \frac{24}{5}\right)}$$