Appendix D: Review of Trigonometry

Measurement of Angles Angles can be measured in degrees or radians. The angle given by a complete revolution contains 360°, or 2π radians.
Thus, $360° = 2\pi$ radians, yielding the conversion formulas

$$\boxed{\left(\frac{180}{\pi}\right)^\circ = 1 \text{ radian}}$$

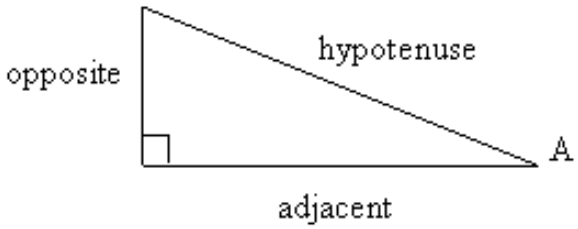
$$\boxed{\frac{\pi}{180} \text{ radians} = 1^\circ}$$

EXAMPLE 1:

a.) Convert 36° to radians.

b.) Convert $-\dfrac{3\pi}{4}$ to degrees.

The Six Trigonometric Functions Consider the right triangle below:



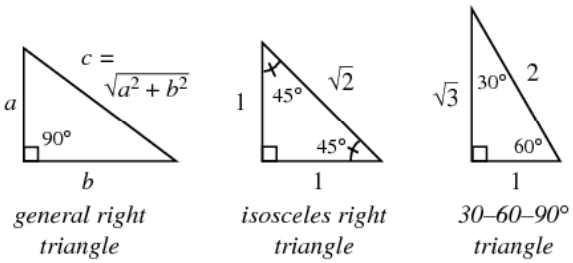
We define

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot A = \frac{\text{adjacent}}{\text{opposite}}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \csc A = \frac{\text{hypotenuse}}{\text{opposite}}$$

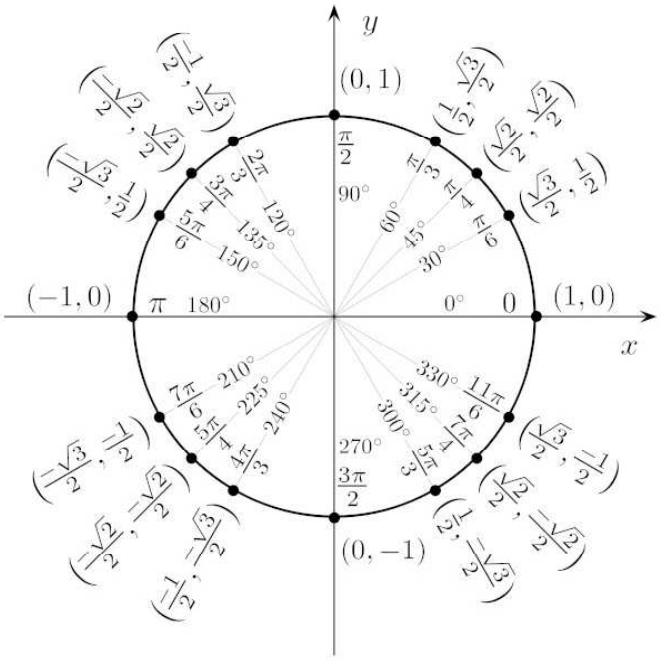
Pythagorean Theorem and two special triangles



general right triangle | *isosceles right triangle* | *30–60–90° triangle*

The Unit Circle

Points of Special Interest on the Unit Circle:
(Download this picture .pdf)



EXAMPLE 2: Find the exact trig ratio's for $\theta = \dfrac{4\pi}{3}$, $\theta = \dfrac{5\pi}{6}$ and $\theta = -\dfrac{\pi}{4}$.

EXAMPLE 3: If $\cos\theta = \frac{1}{5}$, where $0 \leq \theta \leq \frac{\pi}{2}$, find the other 5 trig functions of θ.

Identities to recall

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta)) \qquad \sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$$

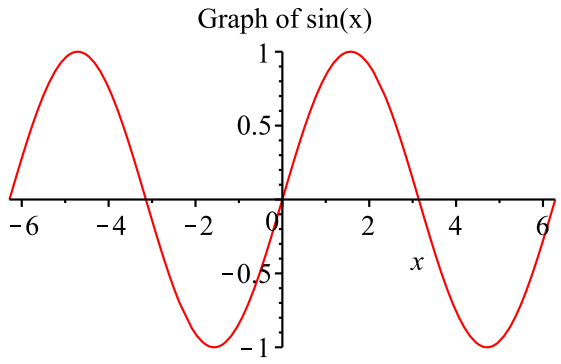
EXAMPLE 4: Solve the following equations for x, where $0 \leq x \leq 2\pi$.

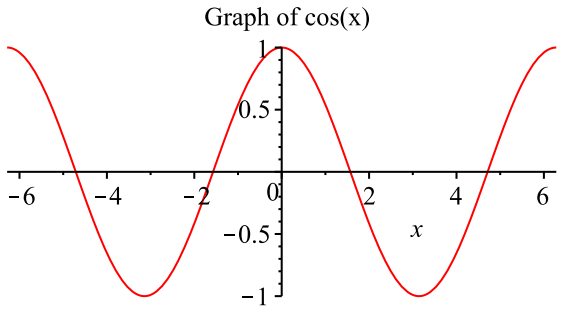
a.) $2\cos x - 1 = 0$

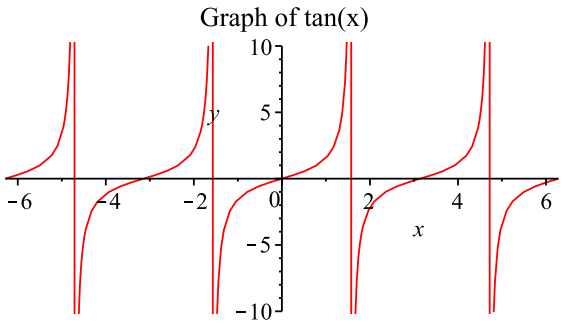
b.) $2\cos x + \sin 2x = 0$

EXAMPLE 5: If $\sec(x) = \frac{5}{3}$, $-\frac{\pi}{2} < x < 0$, find $\sin(2x)$.

Graphs of $\sin x$, $\cos x$, $\tan x$







EXAMPLE 6: Sketch the graph of $f(x) = 1 - \sin x$.

EXAMPLE 7: Sketch the graph of $f(x) = \tan\left(x - \frac{\pi}{2}\right)$.