EXAMPLE 4: Sketch a graph of f satisfying the following conditions:

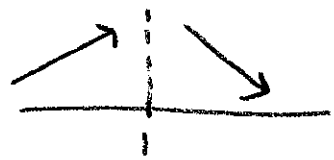
(i) $f'(x) > 0$ on the interval $(-\infty, 1)$ and $f'(x) < 0$ on the interval $(1, \infty)$.

(ii) $f''(x) > 0$ on the interval $(-\infty, -2)$ and $(2, \infty)$.

(iii) $f''(x) < 0$ on the interval $(-2, 2)$.

(iv) $\lim_{x \to -\infty} f(x) = -2$ and $\lim_{x \to \infty} f(x) = 0$.

(i) $f'(x) > 0$ on $(-\infty, 1)$ $\therefore$
$f'(x) < 0$ on $(1, \infty)$

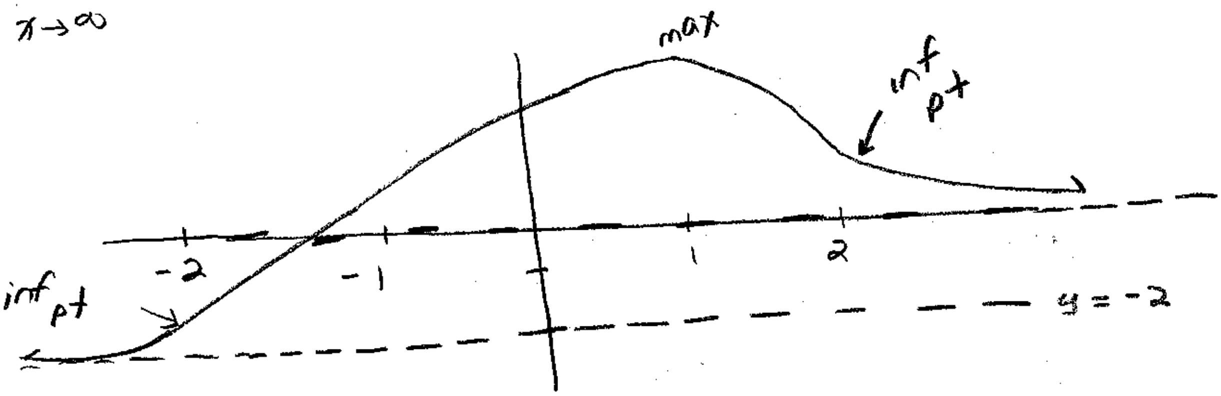


local max at $x = 1$

(ii) $f''(x) > 0$ on $(-\infty, -2) \cup (2, \infty)$ $\therefore$ cu | cd | cu
(iii) $f''(x) < 0$ on $(-2, 2)$ -2 2

inflection points at $x = \pm 2$

(iv) $\lim_{x \to -\infty} f(x) = -2$ $\therefore$ horizontal asymptotes
$\lim_{x \to \infty} f(x) = 0$ at $y = -2$ & $y = 0$



EXAMPLE 5: If $f'(4) = 0$ and $f''(4) = 5$, what can be said about f?

if $f'(4) = 0$ we have a horizontal tangent line at $x = 4$.

$f''(4) = 5 > 0 \Rightarrow f(x)$ is concave up at $x = -4$

$f(x)$ has a local minimum at $x = 4$