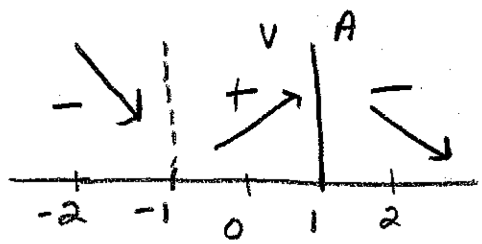
EXAMPLE 6: Sketch the graph of $f(x) = \dfrac{x}{(x-1)^2}$ by locating intervals of increase/decrease, local extrema, concavity and inflection points.

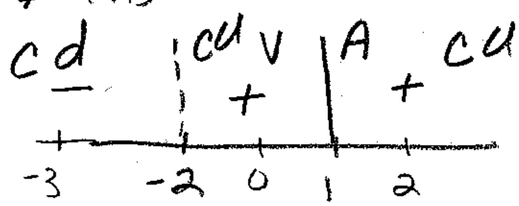
$f(x) = \dfrac{x}{(x-1)^2}$ Vertical Asymptote $x=1$
Horizontal Asymptote $y=0$

$f'(x) = \dfrac{-x-1}{(x-1)^3}$ critical number at $x=-1$

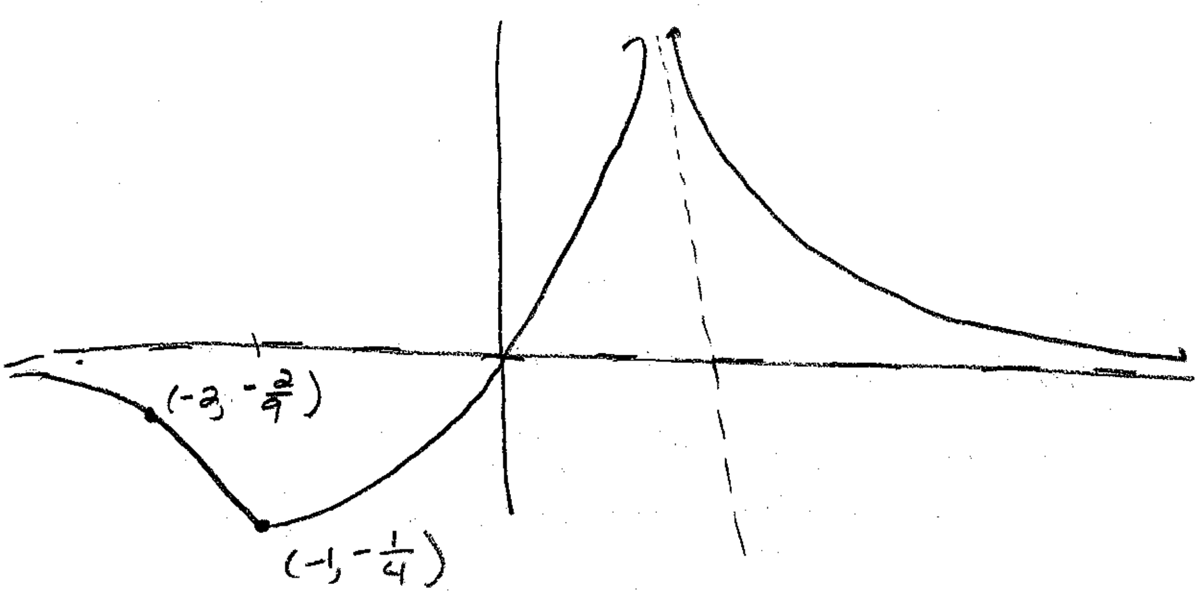


local min at $(-1, -\frac{1}{4})$. No local max since $x=1$ is a V.A.

$f''(x) = \dfrac{2x+4}{(x-1)^4}$ $f''(x) = 0$ if $x=-2$



inflection point: $(-2, -\frac{2}{9})$



Second derivative test for local extrema. If $x = c$ is a critical number for $f(x)$, then:

- If $f''(c) > 0$, then f is concave up, therefore $f(x)$ has a local minimum at $x = c$.
- If $f''(c) < 0$, then f is concave down, therefore $f(x)$ has a local maximum at $x = c$.
- If $f''(c) = 0$ or does not exist, then the test fails, therefore use the first derivative test to find the local extrema.

EXAMPLE 6: Use the second derivative test to find the local extrema for $f(x) = x^3 - 3x - 1$.

EXAMPLE 7: Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that has a local maximum value of 3 at -2 and a local minimum value of 0 at 1.

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f(-2) = 3 \Rightarrow -8a + 4b - 2c + d = 3$

$f(1) = 0 \Rightarrow a + b + c + d = 0$

$f'(-2) = 0 \Rightarrow 12a - 4b + c = 0$

$f'(1) = 0 \Rightarrow 3a + 2b + c = 0$

matrix:

$$\left[\begin{array}{cccc|c} -8 & 4 & -2 & 1 & 3 \\ 1 & 1 & 1 & 1 & 0 \\ 12 & -4 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \end{array}\right]$$

on calculator: choose $\boxed{\text{matrix}} \to \boxed{\text{edit}} \to$ $\underset{\#\text{ rows}}{4} \times \underset{\#\text{ columns}}{5}$

enter fields of matrix.

Go back to home screen

Go to $\boxed{\text{matrix}} \to \boxed{\text{math}}$ scroll to rref,

call up matrix, $\boxed{\text{enter}}$