

Practice Final Exam; to be worked 5/05/10, 8-10:00 am ZACH 102

1. Compute each of the following integrals:

a)  $\int \frac{x-2}{x(x^2+1)} dx$       b)  $\int_{\sqrt{2}}^2 \frac{1}{\sqrt{x^2-1}} dx$       c)  $\int \cos^4(2x) dx$       d)  $\int x \sin(2x) dx$

2. Compute  $\int_0^\infty \left( \frac{2}{2x+1} - \frac{1}{x+3} \right) dx$ .

3. Find the area under the graph of  $y = \sin^3 x$  from  $x = 0$  to  $x = \frac{\pi}{2}$ .

4. The region bounded by  $y = 4 - x^2$  and  $y = 3$  is revolved around the line  $x = 2$ . Find the volume.

5. Find the volume of the solid S whose base is the triangular region with vertices  $(0, 0)$ ,  $(3, 0)$  and  $(0, 4)$  and whose cross sections perpendicular to the  $x$  axis are semicircles.

6.  $\int_0^{\ln 3} \frac{e^x}{\sqrt{e^x+1}} dx =$

7. Find the average value of  $f(x) = \tan x$ ,  $0 \leq x \leq \frac{\pi}{4}$ .

8. A 300 foot rope that weighs 100 pounds hangs over the edge of a building. How much work is done in pulling the first 20 feet of rope to the top?

9. Find the integral that gives area of the surface obtained by rotating the curve  $x = y\sqrt{y}$ ,  $y \in [0, 1]$  about the  $y$ -axis. Do the same for  $x$ -axis revolution.

10. Consider the trough in the shape of a half cylinder of radius 3 feet and length 8 feet (diameter at the top). It is full of water to a depth of 3 feet. Find an integral that gives the work necessary to pump all of the water to a point 1 foot above the top of trough.

11. Approximate  $\int_0^1 e^{-x^2} dx$  to within  $\frac{1}{100}$ .

12. Write a power series for the function  $f(x) = \ln(1 + 2x)$ .

13. A spring stretches 1 foot beyond its natural position under a force of 100 pounds. How much work is done in stretching it 3 feet beyond its natural position?

14. Determine whether the following series converge or diverge. Name the test and apply it completely and correctly.

a)  $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^5+10}}$       b)  $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$       c)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+7}}$       d)  $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{n+1}$

15. Find the second degree Taylor Polynomial for  $f(x) = e^{-x}$  at  $x = 1$ . Using Taylor's Inequality, find an upper bound on the remainder if  $0.5 \leq x \leq 1.1$ .

16. Find the equation of the sphere whose center is at  $(2, -4, 1)$  and which passes through the point  $(-4, 3, -1)$ .

17. Find a unit vector perpendicular to the triangle determined by the points  $P(-1, -5, 2)$ ,  $Q(-4, -2, 5)$ , and  $R(-1, 0, 4)$ .

18. Find the volume of the parallelepiped determined by the vectors  $\langle 1, 0, 2 \rangle$ ,  $\langle 0, 2, 3 \rangle$  and  $\langle 8, -5, 6 \rangle$ .

19. Estimate the error in using the sum of the first 5 terms to approximate  $\sum_{n=1}^{\infty} \frac{1}{n^4}$

20. Find the length of the curve  $y = \sin(3t)$ ,  $x = \cos(3t)$ ,  $0 \leq t \leq \frac{\pi}{12}$ .