Spring 2005 Math 152
Final Exam Practice

(covering Sections 7.1 - 11.3)

courtesy: Amy Austin

NOTE: These problems are to serve merely as practice for your final exam. The final exam for Math 152 is NOT a common exam. Each instructor makes up his or her own final exam. In addition to working this problem set, it is advised that you work the first three exams as well as the night before drills.

1. Find the area bounded by $y = x^2$ and $y = x^3$.
2. Find the area bounded by $x = y^2$ and $x - 2y = 3$.
3. Find the volume of the solid obtained by rotating the region R about the specified axis.

   a.) R is the region bounded by $x = y^2$ and the line $x = 4$. Rotate around the x axis.

   b.) R is the region bounded by $y = x^2$ and $y = 2 - x^2$. Rotate around the x axis.

   c.) R is the region bounded by $y = x$, $x = 3$ and the x axis. Rotate around the y axis.

   d.) R is the region bounded by $y = x - x^2$, $y = 0$. Rotate around the y axis.

   e.) R is the region bounded by $y = x$ and $y = x^2$. Rotate around the line $y = 2$.

   f.) The base of the solid S is the parabolic region $\{(x, y) | y^2 \leq x \leq 1\}$. Cross sections perpendicular to the x axis are squares. Find the volume of S.
4. A rope 100 feet long weighing 2 lbs per foot hangs over a building 100 feet tall. How much work is done in pulling the rope to the top of the building?
5. A right circular cone of height 1 foot has a radius of 1 foot at the top and is filled with a fluid of weight density 60 lbs per cubic foot. How much work is done in pumping the fluid to the top of the tank?
6. A spring stretches 1 foot beyond its natural position under a force of 100 pounds. How much work is done in stretching it 3 feet beyond its natural position?
7. Find the average value of $f(x) = x^3$ over the interval $[1, 3]$
8. Compute $\int_0^1 xe^{2x}\,dx$
9. Compute $\int_1^4 \sqrt{t} \ln t\,dt$
10. Compute $\int_0^{1/2} \cos^{-1} x\,dx$
11. Compute $\int_0^{\pi} \sin^2 x\,dx$
12. Compute $\int \sec^3 x \tan x\,dx$
13. Compute $\int_0^{\pi/4} \tan^2 x \sec^4 x\,dx$
14. Compute $\int_0^{\pi/2} \sin^2 x \cos^3 x\,dx$
15. Compute $\int_0^3 \frac{1}{\sqrt{9 + x^2}}\,dx$
16. Find $\int \frac{\sqrt{x^2 - 4}}{x}\,dx$
17. Compute $\int_0^1 \sqrt{2x - x^2}\,dx$
18. Compute $\int_3^4 \frac{1}{(x-1)(x-2)}\,dx$
19. Find $\int \frac{dx}{x^3 + 2x^2 + x}\,dx$
20. Find $\int \frac{2x}{(x^2+1)(x+1)^2}\,dx$
21. Use the Midpoint rule, Trapezoid rule, and Simpsons rule with $n = 4$ to approximate $\int_1^2 e^{1/x}\,dx$
22. Suppose we use the Trapezoid rule with $n = 6$ to approximate $\int_1^4 \frac{1}{x}\,dx$. Find an upper bound for the error in this approximation.

    FORMULA: $|E_T| \leq \frac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ for $a \leq x \leq b$.
23. Determine whether the following integrals converge or diverge. Evaluate those that converge.

    a.) $\int_1^{\infty} \frac{1}{\sqrt{3x+1}}\,dx$

    b.) $\int_0^1 x^{-2}\,dx$

    c.) $\int_{-\infty}^0 e^{2x}\,dx$

24. Given $y' = e^{x-y}$ and $y(0) = 2$, find $y(1)$

25. A tank contains 100 L of pure water. Brine that contains .1 kg of salt per liter of water enters the tank at a rate of 10 liters per minute. The solution is kept mixed and exits the tank at the same rate. How much salt is in the tank after 6 minutes?

26. Solve $(1 + x^2)y' + 2xy = 3\sqrt{x}$, $y(0) = 2$.

27. Find the length of the curve $y^2 = x^3$, $0 \le x \le \frac{1}{4}$

28. Find the length of the curve $x = t^2$ and $y = t^3$, $1 \le t \le 2$.

29. Find the surface area obtained by revolving the given curve about the specified axis.

    a.) $y = x^2/2$, $0 \le x \le 4$ about the y axis.

    b.) $y = 2x$, $0 \le x \le 1$ about the x axis.

    c.) $y = 2x$, $0 \le x \le 1$ about the y axis.

    d.) $x = t^3$, $y = t^2$, $0 \le t \le 1$ about the x axis.

30. Find the centriod of the region bounded by $y = e^x$, $y = 0$, $x = 0$, $x = 1$.

31. A trough is filled with a liquid of density 840 kg per cubic meter. The ends of the trough are equilateral triangles with sides 8 m long and vertex at the bottom. Find the force on one end of the trough.

32. Determine the limit of the sequence $a_n = \dfrac{5\cos n}{n}$

33. Find the sum of the series:

    a.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

    b.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

    c.) $\displaystyle\sum_{n=2}^{\infty} \ln \frac{n}{n+1}$

34. Determine whether the following series converge or diverge. Be sure to indicate which test you applied.

    a.) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

    b.) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^5+1}}$

    c.) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin^2 n}{5^n}$

    d.) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{\ln n}$

    e.) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n}{(2n-1)!}$

35. Use the third partial sum to approximate the following series. Give an upper bound on the error of the approximation.

    a.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

    b.) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$

36. Find the radius and interval of convergence for the following power series:

    a.) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{\sqrt{n}2^n}$

    b.) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}(x+1)^n}{(2n)!}$

    c.) $\displaystyle\sum_{n=0}^{\infty} n!(3x-2)^n$

37. Find a power series representation for

    $f(x) = \arctan 3x$ and find the radius of convergence.

38. Write $\displaystyle\int_0^{1/2} \frac{\sin x^2}{x}\,dx$ as an infinite series.

39. Find the second degree Taylor polynomial for $f(x) = \sqrt{x}$ at $a = 100$. Using Taylor's Inequality, give an upper bound on the accuracy of this approximation for $99 \le x \le 101$

40. Find a unit vector in the direction of $\mathbf{a} - \mathbf{b}$ where $\mathbf{a} =< 1, 1, 2 >$ and $\mathbf{b} =< 2, 1, 3 >$

41. Find the cross product of the vectors

    $\mathbf{a} =< 1, -2, 4 >$ and $\mathbf{b} =< 2, -3, 0 >$.

42. Find the volume of the parallelipiped determined by $\mathbf{a} =< 1, -2, 4 >$, $\mathbf{b} =< 0, -1, 3 >$, and

    $\mathbf{c} =< 1, 2, -3 >$

43. Given $P(1, 0, 1)$, $Q(0, 2, 3)$ and $R(1, 4, 2)$:

    a.) Find a vector perpendicular to the triangle determined by P, Q and R.

    b.) Find the angle at R

    c.) Find the area of the triangle determined by P, Q and R.