

Practice Final Exam; to be worked 5/06/09, 10:00-noon ZACH 102

1. Compute each of the following integrals:

a) $\int \frac{x-2}{x(x^2+1)} dx$ b) $\int_{\sqrt{2}}^2 \frac{1}{\sqrt{x^2-1}} dx$ c) $\int \cos^4(2x) dx$ d) $\int x \sin(2x) dx$

2. Compute $\int_0^\infty \left(\frac{2}{2x+1} - \frac{1}{x+3} \right) dx$.

3. Find the area under the graph of $y = \sin^3 x$ from $x = 0$ to $x = \frac{\pi}{2}$.

4. The region bounded by $y = 4 - x^2$ and $y = 3$ is revolved around the x axis. Set up, but do not evaluate, integrals to find the volume of the resulting solid using a.) shells and b.) washers.

5. Find the volume of the solid S whose base is the triangular region with vertices $(0, 0)$, $(3, 0)$ and $(0, 4)$ and whose cross sections perpendicular to the x axis are semicircles.

6. Solve the differential equation $\frac{dy}{dx} = \frac{\ln x}{x^3 y^2}$.

7. Find the average value of $f(x) = \tan x$, $0 \leq x \leq \frac{\pi}{4}$

8. A tank contains 1000 L of brine with 100 kg of salt. To increase the concentration, brine with a concentration of 0.25 kg of salt per liter is run into the tank at a rate of 10 L per minute. The thoroughly-mixed solution is drained out at the same rate. Write and solve a differential equation to determine how much salt is in the tank at time t .

9. Find the integral that gives area of the surface obtained by rotating the curve $x = y\sqrt{y}$, $y \in [0, 1]$ about the y -axis. Do the same for x -axis revolution.

10. Consider the trough in the shape of a half cylinder of radius 3 feet and length 8 feet (diameter at the top). It is full of water to a depth of 3 feet.

a.) Find an integral that gives the work necessary to pump all of the water to a point 1 foot above the top of trough.

b.) Find an integral that gives the hydrostatic force of water on the end of the trough (before you pump any water out).

11. A rope 100 feet long weighing 2 lbs per foot hangs over a building 100 feet tall. How much work is done in pulling half the rope to the top of the building?

12. Find the centroid of the region bounded by $y = e^{4x}$, $y = 0$, $x = 0$ and $x = 1$.

13. Write a power series for the function $f(x) = \ln(1 + 2x)$.

14. A spring stretches 1 foot beyond its natural position under a force of 100 pounds. How much work is done in stretching it 3 feet beyond its natural position?

15. Determine whether the following series converge or diverge. Name the test and apply it completely and correctly.

a) $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^5+10}}$ b) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$ c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+7}}$ d) $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{n+1}$

16. Find the second degree Taylor Polynomial for $f(x) = e^{-x}$ at $x = 1$. Using Taylor's Inequality, find an upper bound on the remainder if $0.5 \leq x \leq 1.1$.

17. Find the equation of the sphere whose center is at $(2, -4, 1)$ and which passes through the point $(-4, 3, -1)$.

18. Find a unit vector perpendicular to the triangle determined by the points $P(-1, -5, 2)$, $Q(-4, -2, 5)$, and $R(-1, 0, 4)$.