

Practice Final Exam; to be worked 4/30/08, 10:30-noon ZACH 102

1. Compute each of the following integrals:

a)  $\int \frac{x-2}{x(x^2+1)} dx$       b)  $\int_{\sqrt{2}}^2 \frac{1}{\sqrt{x^2-1}} dx$       c)  $\int \cos^4(2x) dx$       d)  $\int x \sin(2x) dx$

2. Compute  $\int_0^\infty \left( \frac{2}{2x+1} - \frac{1}{x+3} \right) dx$ .

3. Find the area under the graph of  $y = \sin^3 x$  from  $x = 0$  to  $x = \frac{\pi}{2}$ .

4. The region bounded by  $y = 4 - x^2$  and  $y = 3$  is revolved around the  $x$  axis. Set up, but do not evaluate, integrals to find the volume of the resulting solid using a.) shells and b.) washers.

5. Find the volume of the solid S whose base is the triangular region with vertices  $(0, 0)$ ,  $(3, 0)$  and  $(0, 4)$  and whose cross sections perpendicular to the  $x$  axis are semicircles.

6. Solve the differential equation  $\frac{dy}{dx} = \frac{\ln x}{x^3 y^2}$ .

7. Find the average value of  $f(x) = \tan x$ ,  $0 \leq x \leq \frac{\pi}{4}$

8. A tank contains 1000 L of brine with 100 kg of salt. To increase the concentration, brine with a concentration of 0.25 kg of salt per liter is run into the tank at a rate of 10 L per minute. The thoroughly-mixed solution is drained out at the same rate. Write and solve a differential equation to determine how much salt is in the tank at time  $t$ .

9. Find the integral that gives area of the surface obtained by rotating the curve  $x = y\sqrt{y}$ ,  $y \in [0, 1]$  about the  $y$ -axis. Do the same for  $x$ -axis revolution.

10. Consider the trough in the shape of a half cylinder of radius 3 feet and length 8 feet (diameter at the top). It is full of water to a depth of 3 feet.

a.) Find an integral that gives the work necessary to pump all of the water to a point 1 foot above the top of trough.

b.) Find an integral that gives the hydrostatic force of water on the end of the trough (before you pump any water out).

11. Find the centroid of the region bounded by  $y = e^{4x}$ ,  $y = 0$ ,  $x = 0$  and  $x = 1$ .

12. Write a power series for the function  $f(x) = \ln(1 + 2x)$ .

13. A spring stretches 1 foot beyond its natural position under a force of 100 pounds. How much work is done in stretching it 3 feet beyond its natural position?

14. Determine whether the following series converge or diverge. Name the test and apply it completely and correctly.

a)  $\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^5+10}}$       b)  $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$       c)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+7}}$       d)  $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{n+1}$

15. Find the second degree Taylor Polynomial for  $f(x) = e^{-x}$  at  $x = 1$ . Using Taylor's Inequality, find an upper bound on the remainder if  $0.5 \leq x \leq 1.1$ .

16. Find the equation of the sphere whose center is at  $(2, -4, 1)$  and which passes through the point  $(-4, 3, -1)$ .

17. Find a unit vector perpendicular to the triangle determined by the points  $P(-1, -5, 2)$ ,  $Q(-4, -2, 5)$ , and  $R(-1, 0, 4)$ .