

**MATH 152  
SPRING 2008**

**SAMPLE EXAM II**

**Part I - Multiple Choice**

1. The velocity of a rocket (in meters per second) is measured at 2 second intervals in the chart below. Approximate the distance the rocket travelled from  $t = 0$  to  $t = 8$  using the midpoint rule with  $n = 2$ .

|          |   |   |    |    |    |
|----------|---|---|----|----|----|
| time     | 0 | 2 | 4  | 6  | 8  |
| velocity | 0 | 5 | 12 | 30 | 70 |

- a) 99 m                      b) 188 m                      c) 164 m  
d) 156 m                      e) 140 m

2. The integral  $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$

- a) converges to 0  
b) diverges to  $\infty$   
c) diverges by oscillation  
d) converges by comparison to  $\int_1^{\infty} \frac{1}{x^2} dx$   
e) diverges by comparison to  $\int_1^{\infty} \frac{1}{x} dx$

3.  $\int_0^{\infty} x e^{-x^2/2} dx =$

- a) divergent                      b)  $\frac{1}{2}$                       c) 0  
d)  $\frac{1}{4}$                       e) 1

4. Find a general solution to the differential equation  $\frac{dy}{dx} = 3y + 6$ .

a)  $y = 2 + Ce^{-3x}$

b)  $y = -18 + Ce^{3x}$

c)  $y = -2 + Ce^{3x}$

d)  $y = 18 + Ce^{-3x}$

e) None of these.

5. Which is an integrating factor for the differential equation  $\frac{dy}{dt} - y \tan t = 1$ ?

a)  $\cos t$

b)  $e^{-\tan t}$

c)  $\tan t$

d)  $\sec t$

e)  $e^{-\sec^2 t}$

6. Find the length of the curve  $y = 4x^{3/2}$  from  $(0, 0)$  to  $(2, 2^{(7/2)})$ .

a)  $\frac{1}{54}(73\sqrt{73} - 1)$

b)  $\frac{1}{27}(73\sqrt{73} - 1)$

c)  $\frac{1}{54}(37\sqrt{37} - 1)$

d)  $\frac{1}{27}(37\sqrt{37} - 1)$

e) None of these.

7. Find the length of the curve  $x = t^3, y = t^2, 0 \leq t \leq 1$ .

a)  $\frac{1}{27}(13\sqrt{13} - 8)$

b)  $\frac{2}{27}(13\sqrt{13} - 8)$

c)  $\frac{2}{17}(23\sqrt{23} - 16)$

d)  $\frac{1}{27}(23\sqrt{23} - 8)$

e)  $\frac{1}{27}(23\sqrt{23} - 16)$

8. Which of the following integrals gives the area of the surface obtained by rotating the curve  $y = e^{2x}$ ,  $0 \leq x \leq 1$  about the  $x$ -axis?

a)  $\int_0^1 2\pi\sqrt{1+4e^{4x}} dx$

b)  $\int_0^1 2\pi\sqrt{1+\frac{1}{4}e^{4x}} dx$

c)  $\int_0^1 2\pi e^{2x}\sqrt{1+2e^{2x}} dx$

d)  $\int_0^1 2\pi e^{2x}\sqrt{1+4e^{4x}} dx$

e)  $\int_0^1 2\pi x\sqrt{1+4e^{4x}} dx$

9. Find the center of mass of the system with masses 10g, 4g, and 2g located at the points  $(1, 3)$ ,  $(-2, 5)$  and  $(1, -6)$ , respectively.

a)  $(\frac{19}{8}, \frac{1}{4})$

b)  $(4, \frac{8}{19})$

c)  $(\frac{8}{19}, 4)$

d)  $(\frac{1}{4}, \frac{19}{8})$

e) None of the above.

10. A rectangular tank with width 4m, length 25m and height 3m is full of water of density  $\rho g$ . Find the hydrostatic force on the bottom of the tank.

a)  $(300)\rho g$  N

b)  $(200)\rho g$  N

c)  $(150)\rho g$  N

d)  $(250)\rho g$  N

e)  $(310)\rho g$  N

## Part II - Work Out Problems

11. In using Simpson's rule with  $n$  intervals to approximate  $\int_a^b f(x) dx$ , the error is at most  $\frac{K(b-a)^5}{180n^4}$ . Using this estimate, find an expression to determine the smallest value of  $n$  to guarantee that the Simpson's rule approximation to  $\int_1^3 \ln x dx$  is within .000001 of the exact answer. (NOTE: You do NOT have to simplify your expression to an exact integer!)

12. Solve the initial value problem  $\frac{dy}{dx} = x^2(1 + y^2)$ ,  $y(0) = 1$  explicitly for  $y$ .

13. A tank originally contains 100 L of pure water. Water containing 2 kg of salt per liter enters the tank at a rate of 2 L/min. The mixture is thoroughly stirred and leaves the tank at the same rate. Write and solve an initial value problem to find the amount of salt in the tank at any time  $t$ .

14. Find the surface area obtained by rotating the curve parametrized by  $x = \cos^2 t$ ,  $y = \sin^2 t$ ,  $0 \leq t \leq \frac{\pi}{2}$  about the  $y$  axis.

15. Find the centroid of the region bounded by  $y = e^x$ ,  $x = 0$ ,  $x = 1$ ,  $y = 0$ .

16. The ends of a water tank are vertical and are shaped as the region bounded by  $y = 4x^2$  and  $y = 4$ . Find the hydrostatic force against the end of the tank if the tank is full. The density of water is  $\rho = 1000 \text{ kg/m}^3$ , and use  $g = 9.8 \text{ m/s}^2$ .