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MATH 251 Fall 2018
EXAM IV - VERSION A

LAST NAME: ______________ FIRST NAME: ______________

SECTION NUMBER: ________

UIN: ______________

note: your final exam is all mc!

DIRECTIONS:

1. This is a NON calculator exam.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 11-17), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the **quality** and **correctness** of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________

DO NOT WRITE BELOW!

Question Type	Points Awarded	Points
Multiple Choice		40
Free Response		60
Total		100

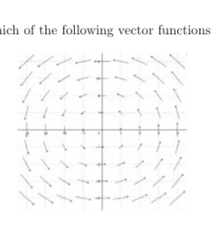
PART I: Multiple Choice. 4 points each.

1. Use the divergence theorem to evaluate $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F} = \langle x,\ e^{\sin(2z)},\ (5+3y^{200})^7\rangle$ and S is the surface bounded by $x^2+y^2=4$, $z=0$, $z=5$.
   (a) 10π
   (b) 6π
   (c) 0
   (d) 4π
   (e) 20π KEY

2. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \langle 3+2xy^2,\ 2x^2y\rangle$ and C parameterized by $\mathbf{r}(t) = \langle t, t^2\rangle$, $-1 \le t \le 1$.
   (a) 6 KEY
   (b) 5
   (c) -5
   (d) 2
   (e) -6

3. Evaluate $\int_C y\sin z\, ds$, where C is parameterized by $x=\cos t$, $y=\sin t$, $z=t$, $0\le t\le 2\pi$.
   (a) $-\pi\sqrt{2}$
   (b) $\pi\sqrt{2}$ KEY
   (c) π
   (d) 0
   (e) $-\pi$

4. Which of the following vector functions $\mathbf{F}$ has the vector field shown below?



   (a) $\mathbf{F} = \langle x, y\rangle$
   (b) $\mathbf{F} = \langle -y, x\rangle$ KEY
   (c) $\mathbf{F} = \langle y, x\rangle$
   (d) $\mathbf{F} = \langle -x, -y\rangle$
   (e) None of the above

5. Find the surface area of the part of the surface $z = xy$ that lies inside the cylinder $x^2+y^2=1$.
   (a) $\frac{4\pi}{3}\sqrt{2}$
   (b) $\frac{4\pi}{3}\left(2\sqrt{2}-1\right)$
   (c) $\frac{4\pi}{3}$
   (d) $\frac{2\pi}{3}\left(2\sqrt{2}-1\right)$ KEY
   (e) $\frac{4\pi}{3}\left(2\sqrt{2}+1\right)$

6. Find the work done by the force field $\mathbf{F} = \left\langle 8y+7e^{\sqrt{x}}, 8x+9\cos(y^2)\right\rangle$ where C is the triangle with vertices $(0,0)$, $(1,5)$ and $(2,1)$.
   (a) 1
   (b) $\frac{2}{3}$
   (c) 0 KEY
   (d) $\frac{5}{3}$
   (e) None of these.

7. Find the flux of $\mathbf{F} = \langle x,\ y,\ -z\rangle$ across S, where S is the part of the paraboloid $z = 4-x^2-y^2$ that is above the xy-plane. Use the positive (upward) orientation.
   (a) 0
   (b) 6π
   (c) -8π
   (d) -6π
   (e) 8π KEY

8. A particle starts at the point $(-3,0)$, moves along the x-axis to the point $(3,0)$, then along the top half of the circle $x^2+y^2=9$. Using Green's Theorem, find the work done on this particle by the force field $\mathbf{F} = \langle 3x, x^3+3xy^2\rangle$.
   (a) $\frac{243\pi}{4}$ KEY
   (b) $\frac{243\pi}{2}$
   (c) 81π
   (d) 27π
   (e) 54π

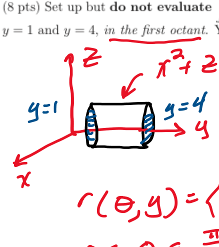
9. Evaluate $\int_C (z+xy)\,ds$, where C is the line segment from $(-1,1,0)$ to $(1,2,0)$.
   (a) $\frac{5\sqrt{5}}{3}$
   (b) $\frac{1}{6}$
   (c) $\frac{\sqrt{5}}{6}$ KEY
   (d) $\frac{\sqrt{5}}{2}$
   (e) $\frac{5}{6}$

10. Which of the following is the correct double integral used in evaluating $\iint_S (x^2+y^2)\, dS$ where S is the hemisphere $x^2+y^2+z^2=4$, $z\ge 0$?
   Note: If we parameterize the sphere $x^2+y^2+z^2=\rho^2$ by $\mathbf{r}(\theta,\phi) = \langle \rho\sin(\phi)\cos(\theta),\ \rho\sin(\phi)\sin(\theta),\ \rho\cos(\phi)\rangle$, then $|\mathbf{r}_\theta\times\mathbf{r}_\phi| = \rho^2\sin(\phi)$.
   (a) $\int_0^{2\pi}\int_0^{\pi/2} (16\sin^3\phi\cos\phi)\, d\phi\, d\theta$
   (b) $\int_0^{2\pi}\int_0^{\pi/2} (32\sin^3\phi\cos\phi)\, d\phi\, d\theta$
   (c) $\int_0^{2\pi}\int_0^{\pi} (16\sin^3\phi)\, d\phi\, d\theta$
   (d) $\int_0^{2\pi}\int_0^{\pi} (32\sin^3\phi)\, d\phi\, d\theta$
   (e) $\int_0^{2\pi}\int_0^{\pi/2} (16\sin^3\phi)\, d\phi\, d\theta$ KEY

Part II: Work out. Show all appropriate work.

11. (10 pts) Let f be a scalar field and let $\mathbf{F}$ be a vector field. Determine whether the following statements are true or false. Write "True" or "False" in the blank provided. No work needs to be shown. Two points each.
   (a) __T__ The curl of the gradient of f is meaningful and results in a vector.
   (b) __F__ The divergence of $\mathbf{F}$ is a vector field. scalar field
   (c) __F__ div(div $\mathbf{F}$) is meaningful and results in a scalar. meaningless
   (d) __F__ The gradient of the divergence of $\mathbf{F}$ is meaningless. vector field
   (e) __T__ If f has continuous partial derivatives and C is any circle, then $\int_C \nabla f\cdot d\mathbf{r} = 0$.
   f is conservative → line integral over closed curve = 0

12. (8 pts) Set up but **do not evaluate** $\iint_S xyz\, dS$, where S is the part of the cylinder $x^2+z^2=9$ between the planes $y=1$ and $y=4$, *in the first octant.* Your limits of integration must be defined with the appropriate differential.



$r(\theta,y) = \langle 3\cos\theta, y, 3\sin\theta\rangle$

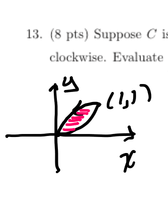
$0\le\theta\le\frac{\pi}{2}$, $1\le y\le 4$

$r_\theta\times r_y = \begin{vmatrix} i & j & k \\ -3\sin\theta & 0 & 3\cos\theta \\ 0 & 1 & 0 \end{vmatrix} = \langle -3\cos\theta, 0, -3\sin\theta\rangle$

$|r_\theta\times r_y| = 3$

$\iint_S xyz\, ds = \int_0^{\frac{\pi}{2}}\int_1^4 9\cos\theta\sin\theta\, y\cdot 3\, dy\, d\theta$

13. (8 pts) Suppose C is the boundary of the region enclosed by the parabolas $y=x^2$ and $x=y^2$, oriented counter-clockwise. Evaluate $\oint_C (y+e^{\sqrt{x}})dx + (4x+\cos(y^2))dy$. Simplify.



Green's: $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D (4-1)\, dA$

$\iint_D 3\, dA$ D: $x^2\le y\le\sqrt{x}$, $0\le x\le 1$

$\int_0^1\int_{x^2}^{\sqrt{x}} 3\, dy\, dx = \int_0^1 (3\sqrt{x}-3x^2)\, dx$

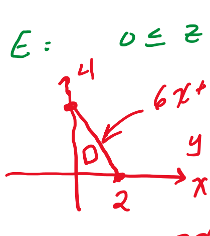
$= \left(3\cdot\frac{2}{3}x^{3/2} - x^3\right)\Big|_0^1 = \boxed{1}$

14. (10 pts) Use the The Divergence Theorem to set up but **do not evaluate** the integral that gives the flux of $\mathbf{F} = \langle x^2y,\ xy^2,\ 3xyz\rangle$ across S, where S is the tetrahedron bounded by the three coordinate planes and the plane $6x+3y+z=12$.

Flux F $= \iint_S F\cdot ds = \iiint_E \text{div}\, F\, dV = \iiint_E (2xy+2xy+3xy)\, dV$

$= \iiint_E 7xy\, dV$

E: $0\le z\le 12-6x-3y$



$= \iint_R \left(\int_0^{12-6x-3y} 7xy\, dz\right) dA$

D: $0\le y\le 4-2x$
$0\le x\le 2$

$= \int_0^2\int_0^{4-2x}\int_0^{12-6x-3y} 7xy\, dz\, dy\, dx$

15. (8 pts) Use Stokes' Theorem to set up but **do not evaluate** $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \langle xy, 5z, 7y\rangle$ and where C is the intersection of the plane $x+z=9$ and the cylinder $x^2+y^2=81$. Your limits of integration must be defined with the appropriate differential. Assume C is oriented counterclockwise when looking from above.

$\int_C F\cdot dr = \iint_S \text{curl}\, F\cdot ds$

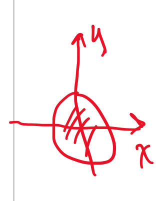
curl $F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & 5z & 7y \end{vmatrix} = \langle 2, 0, -x\rangle$

S: $x+z=9$, where $x^2+y^2\le 81$

$r(x,y) = \langle x, y, 9-x\rangle$, $x^2+y^2\le 81$

$r_x\times r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} = \langle 1, 0, 1\rangle$

$\iint_S \text{curl} F\cdot dS = \iint_D \text{curl} F(r(x,y))\cdot(r_x\times r_y)\, dA$



$= \iint_{x^2+y^2=81} \langle 2, 0, x\rangle\cdot\langle 1, 0, 1\rangle\, dA$ ↳ $2+x$, in polar

$\int_0^{2\pi}\int_0^9 (2+r\cos\theta)\, r\, dr\, d\theta$

16. (8 pts) Evaluate $\int_C 4xy\, dx + (3x+6y)\, dy$ where C is the path of the parabola $y=x^2$ from the point $(1,1)$ to the point $(2,4)$. Simplify.

$r(t) = \langle t, t^2\rangle$, $1\le t\le 2$ $x=t$, $y=t^2$; $dx=dt$, $dy=2t\,dt$

$\int_0^1 4\cdot t\cdot t^2\cdot dt + (3t+6t^2)(2t)\, dt$

$\int_0^1 (4t^3+6t^2+12t^3)\, dt$

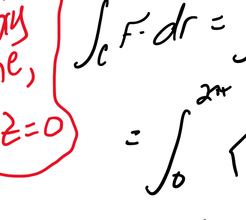
$\int_0^1 (16t^3+6t^2)\, dt = 4t^4+2t^3\Big|_0^1$

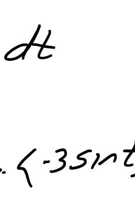
$= \boxed{6}$

17. (8 pts) Use Stokes' theorem to evaluate $\iint_S \text{curl}\,\mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = \langle 6y\cos z, \sin z, x\rangle$ and S is the hemisphere $x^2+y^2+z^2=9$, $z\ge 0$, oriented upward.

Stokes: $\iint_S \text{curl} F\, dS = \int_C F\cdot dr$, C = boundary curve of S

C: $x^2+y^2=9$





$r(t) = \langle 3\cos t, 3\sin t, 0\rangle$, $0\le t\le 2\pi$

$r'(t) = \langle -3\sin t, 3\cos t, 0\rangle$

circle in xy plane, so z=0

$\int_C F\cdot dr = \int_0^{2\pi} F(r(t))\cdot r'(t)\, dt$

$= \int_0^{2\pi} \langle 6\cdot 3\sin t, 0, 3\cos t\rangle\cdot\langle -3\sin t, 3\cos t, 0\rangle\, dt$

$\int_0^{2\pi} -54\sin^2 t\, dt = -54\int_0 \frac{1}{2}(1-\cos 2t)\, dt$

$= -27\left(t - \frac{1}{2}\sin 2t\right)\Big|_0^{2\pi}$

$= -27(2\pi)$