

MATH 251 Spring 2017  
EXAM I - VERSION A

LAST NAME: \_\_\_\_\_ FIRST NAME: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

UIN: \_\_\_\_\_

**DIRECTIONS:**

1. You may use a calculator on this exam.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 11-16), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the **quality** and **correctness** of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form*.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: \_\_\_\_\_

**DO NOT WRITE BELOW!**

Question Type	Points Awarded	Points
Multiple Choice		40
Free Resepose		60
Total		100

**PART I: Multiple Choice. 4 points each.**

**Part I** Multiple choice. (4 points each).

1. The equations  $x^2 + z^2 = 4$  and  $y = -1$  represent what in  $\mathbb{R}^3$ ?

- (a) A cylinder centered along the  $z$  axis
- (b) A circle centered along the  $z$  axis
- (c) A cylinder centered along the  $y$  axis
- (d) A circle centered along the  $y$  axis
- (e) A sphere

2. Which of the following is the domain of the vector function  $\mathbf{r}(t) = \left\langle \frac{t}{\ln(t-1)}, \sqrt{9-t^2}, e^t \right\rangle$ ?

- (a)  $(1, 2) \cup (2, 3]$
- (b)  $(1, 2) \cup (2, 3)$
- (c)  $(1, 3]$
- (d)  $[-3, 3]$
- (e)  $(-3, 3)$

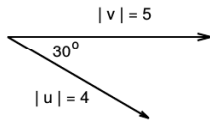
3. Find  $\lim_{t \rightarrow 0} \left\langle \frac{t}{t^2 + 3t}, \frac{\sin(\pi t)}{\ln(t+1)}, \frac{1}{2^t} \right\rangle$

- (a)  $\langle 0, -1, 1 \rangle$
- (b)  $\left\langle \frac{1}{3}, \pi, 1 \right\rangle$
- (c)  $\langle 0, \pi, 1 \rangle$
- (d)  $\left\langle \frac{1}{3}, 1, 1 \right\rangle$
- (e)  $\left\langle \frac{1}{3}, -\pi, 1 \right\rangle$

4. Find the scalar projection of  $\langle -5, 3, 2 \rangle$  onto  $\mathbf{i} - \mathbf{k}$ .

- (a)  $-\frac{8}{\sqrt{2}}$
- (b)  $-\frac{5}{\sqrt{2}}$
- (c)  $-\frac{7}{\sqrt{38}}$
- (d)  $-\frac{8}{\sqrt{38}}$
- (e)  $-\frac{7}{\sqrt{2}}$

5. Find  $|\mathbf{v} \times \mathbf{u}|$  and whether  $\mathbf{v} \times \mathbf{u}$  points in or out of the page.



- (a)  $|\mathbf{v} \times \mathbf{u}| = 10\sqrt{3}$  and points out of the page
  - (b)  $|\mathbf{v} \times \mathbf{u}| = 10$  and points into the page
  - (c)  $|\mathbf{v} \times \mathbf{u}| = 10\sqrt{3}$  and points into the page
  - (d)  $|\mathbf{v} \times \mathbf{u}| = 10$  and points out of the page
  - (e) None of these
6. Find the equation of the plane that contains the line  $x = 1 + t$ ,  $y = 2 - t$  and  $z = 4 - 3t$  and is parallel to the plane  $5x + 2y + z = 1$ .
- (a)  $-x + 2y + z - 7 = 0$
  - (b)  $5x + 2y + z - 13 = 0$
  - (c)  $-x + 2y + z + 6 = 0$
  - (d)  $5x + 2y + z - 9 = 0$
  - (e)  $5x + 2y + z - 7 = 0$

7. Find the angle at vertex  $B$  for  $\triangle ABC$ .  $A(1, 0, -1)$ ,  $B(3, -2, 0)$  and  $C(1, 3, 3)$ .

(a)  $\arccos\left(\frac{11}{3\sqrt{38}}\right)$

(b)  $\arccos\left(\frac{2}{15}\right)$

(c)  $\arccos\left(\frac{-11}{3\sqrt{38}}\right)$

(d)  $\arccos\left(\frac{-2}{15}\right)$

(e) None of the above

8. The space curve shown here has which equation? Assume  $t \geq 0$ .

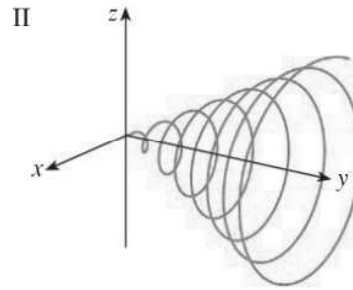
(a)  $\mathbf{r}(t) = \langle t \cos t, t, t \sin t \rangle$

(b)  $\mathbf{r}(t) = \left\langle \cos t, \sin t, \frac{1}{1+t^2} \right\rangle$

(c)  $\mathbf{r}(t) = \left\langle t, \frac{1}{1+t^2}, t^2 \right\rangle$

(d)  $\mathbf{r}(t) = \langle \cos 8t, \sin 8t, e^t \rangle$

(e)  $\mathbf{r}(t) = \langle \cos t, \cos 3t, \cos 4t \rangle$



9. Find the equation of the line passing through the point  $P(1, 2, 3)$  that is perpendicular to both  $\langle 3, -1, 2 \rangle$  and  $\langle 1, 0, -2 \rangle$ .

(a)  $x = 2 + t, y = 8 + 2t, z = 1 + 3t$

(b)  $x = 2 + t, y = -8 + 2t, z = 1 + 3t$

(c)  $x = 1 + 2t, y = 2 + 8t, z = 3 + t$

(d)  $x = 1 + 2t, y = 2 - 8t, z = 3 + t$

(e)  $x = 2 - t, y = -8 - 2t, z = 1 - 3t$

10. What is the intersection of the sphere  $x^2 + y^2 + z^2 + 6x - 4y - 10z - 26 = 0$  with the  $yz$  plane?

(a) The  $yz$  plane intersects the sphere in the circle  $(y - 2)^2 + (z - 5)^2 = 55$

(b) The  $yz$  plane intersects the sphere at the point  $(-3, 2, 5)$

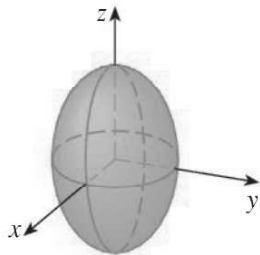
(c) The  $yz$  plane intersects the sphere in the circle  $(y - 2)^2 + (z - 5)^2 = 73$

(d) The  $yz$  plane does not intersect the sphere.

(e) The  $yz$  plane intersects the sphere in the circle  $(y - 2)^2 + (z - 5)^2 = 26$

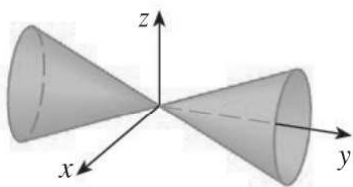
**Part II:** Work out. Show all intermediate steps.

11. (10 pts) Match the graph with its equation by putting the corresponding letter in the blank provided.



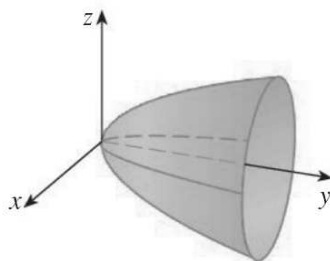
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A.  $\frac{x^2}{4} - \frac{y^2}{9} + \frac{z^2}{9} = 1$



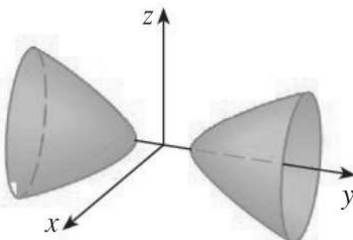
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B.  $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} = 1$



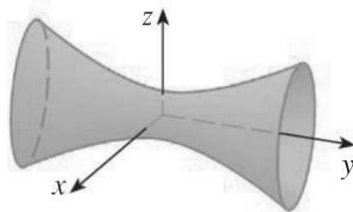
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C.  $y = \frac{x^2}{4} + \frac{z^2}{9}$



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D.  $-\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{9} = 1$



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E.  $\frac{y^2}{4} = \frac{x^2}{9} + \frac{z^2}{9}$

12. (10 pts) Given that the vector functions  $\mathbf{r}_1(t) = \langle -2t, t^5, -5t^3 \rangle$  and  $\mathbf{r}_2(u) = \langle \sin(-2u), \sin(u), u - \pi \rangle$  intersect at the origin, find the angle of intersection, rounded to the nearest degree.

13. Consider the planes  $x - 3y + z = 11$  and  $x - 2y - z = 4$ .

a.) (4 pts) Find the point where the line of intersection of these two planes passes through the  $xy$ -plane.

b.) (4 pts) Find a vector,  $\mathbf{v}$ , that is parallel to the line of intersection of these two planes.

c.) (6 pts) Using the information in parts a.) and b.), find a **parametric equation** for the line of intersection of these planes.

14. (8 pts) Find parametric equations for the tangent line to the curve  $x = t^2 + t$ ,  $y = 4\sqrt{t}$ ,  $z = e^{t^3-t}$  at  $(2, 4, 1)$ .

15. (8 pts) Find the length of the curve  $\mathbf{r}(t) = \langle t\sqrt{2}, e^t, e^{-t} \rangle$  for  $0 \leq t \leq 3$ .

16. (10 pts) Find  $\mathbf{r}(t)$  if  $\mathbf{r}'(t) = \langle t, e^t, 2t \ln t \rangle$  and  $\mathbf{r}(1) = \langle 2, -3, 1 \rangle$ . In order to get full credit, steps must be shown if a technique of integration is needed to find the antiderivative.