MATH 251 Spring 2017
EXAM I - VERSION A

LAST NAME: Key FIRST NAME: ____________________

SECTION NUMBER: ____________

UIN: ______________________________

DIRECTIONS:

1. You may use a calculator on this exam.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 11-16), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

DO NOT WRITE BELOW!

Question Type	Points Awarded	Points
Multiple Choice		40
Free Resepose		60
Total		100

PART I: Multiple Choice. 4 points each.

Part I Multiple choice. (4 points each).

1. The equations $x^2 + z^2 = 4$ and $y = -1$ represent what in $\mathbb{R}^3$?

(a) A cylinder centered along the z axis

(b) A circle centered along the z axis

(c) A cylinder centered along the y axis

(d) A circle centered along the y axis

(e) A sphere

2. Which of the following is the domain of the vector function $\mathbf{r}(t) = \left\langle \frac{t}{\ln(t-1)}, \sqrt{9-t^2}, e^t \right\rangle$?

(a) $(1,2) \cup (2,3]$

(b) $(1,2) \cup (2,3)$

(c) $(1,3]$

(d) $[-3,3]$

(e) $(-3,3)$

$t \neq 2,\ t > 1,\ -3 \leq t \leq 3$

$(1,2) \cup (2,3)$

3. Find $\lim_{t \to 0} \left\langle \frac{t}{t^2+3t}, \frac{\sin(\pi t)}{\ln(t+1)}, \frac{1}{2^t} \right\rangle$ $= \lim_{t \to 0} \left\langle \frac{\cancel{t}}{\cancel{t}(t+3)}, \frac{\pi \cos(\pi t)}{\frac{1}{t+1}}, \frac{1}{2^t} \right\rangle$

$= \left\langle \frac{1}{3}, \pi, 1 \right\rangle$

(a) $\langle 0, -1, 1 \rangle$

(b) $\left\langle \frac{1}{3}, \pi, 1 \right\rangle$

(c) $\langle 0, \pi, 1 \rangle$

(d) $\left\langle \frac{1}{3}, 1, 1 \right\rangle$

(e) $\left\langle \frac{1}{3}, -\pi, 1 \right\rangle$

4. Find the scalar projection of $\langle -5, 3, 2\rangle$ onto $\mathbf{i} - \mathbf{k}$.

(a) $-\frac{8}{\sqrt{2}}$

(b) $-\frac{5}{\sqrt{2}}$

(c) $-\frac{7}{\sqrt{38}}$

(d) $-\frac{8}{\sqrt{38}}$

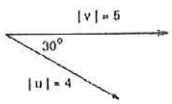
(e) $-\frac{7}{\sqrt{2}}$

$b = \langle -5, 3, 2 \rangle$

$\vec{a} = \langle 1, 0, -1 \rangle$

$\text{comp}_a b = \frac{-5-2}{\sqrt{2}}$

5. Find $|\mathbf{v} \times \mathbf{u}|$ and whether $\mathbf{v} \times \mathbf{u}$ points in or out of the page.



(a) $|\mathbf{v} \times \mathbf{u}| = 10\sqrt{3}$ and points out of the page

(b) $|\mathbf{v} \times \mathbf{u}| = 10$ and points into the page

(c) $|\mathbf{v} \times \mathbf{u}| = 10\sqrt{3}$ and points into the page

(d) $|\mathbf{v} \times \mathbf{u}| = 10$ and points out of the page

(e) None of these

$\bar{v} \times u = |\bar{v}||u| \sin 30^\circ$

$= 20 \cdot \frac{1}{2}$

$= 10$

RHR $\bar{v} \times u$ points into page

6. Find the equation of the plane that contains the line $x = 1 + t$, $y = 2 - t$ and $z = 4 - 3t$ and is parallel to the plane $5x + 2y + z = 1$.

(a) $-x + 2y + z - 7 = 0$

(b) $5x + 2y + z - 13 = 0$

(c) $-x + 2y + z + 6 = 0$

(d) $5x + 2y + z - 9 = 0$

(e) $5x + 2y + z - 7 = 0$

let $t = 0 \rightarrow (1, 2, 4)$

$\vec{n} = \langle 5, 2, 1 \rangle$

$\langle 5, 2, 1 \rangle \cdot \langle x-1, y-2, z-4 \rangle = 0$

$5x - 5 + 2y - 4 + z - 4 = 0$

$5x + 2y + z - 13 = 0$

7. Find the angle at vertex B for ΔABC. $A(1,0,-1)$, $B(3,-2,0)$ and $C(1,3,3)$.

(a) $\arccos\left(\frac{11}{3\sqrt{38}}\right)$

(b) $\arccos\left(\frac{2}{15}\right)$

(c) $\arccos\left(\frac{-11}{3\sqrt{38}}\right)$

(d) $\arccos\left(\frac{-2}{15}\right)$

(e) None of the above

$BA = \langle -2, 2, -1 \rangle$

$BC = \langle -2, 5, 3 \rangle$

$\cos\theta = \frac{4+10-3}{3\sqrt{38}}$

$\cos\theta = \frac{11}{3\sqrt{38}}$

8. The space curve shown here has which equation? Assume $t \geq 0$.

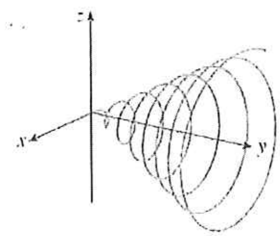
(a) $\mathbf{r}(t) = \langle t\cos t, t, t\sin t \rangle$

(b) $\mathbf{r}(t) = \left\langle \cos t, \sin t, \frac{1}{1+t^2} \right\rangle$

(c) $\mathbf{r}(t) = \left\langle t, \frac{1}{1+t^2}, t^2 \right\rangle$

(d) $\mathbf{r}(t) = \langle \cos 8t, \sin 8t, e^t \rangle$

(e) $\mathbf{r}(t) = \langle \cos t, \cos 3t, \cos 4t \rangle$



9. Find the equation of the line passing through the point $P(1,2,3)$ that is perpendicular to both $\langle 3,-1,2 \rangle$ and $\langle 1,0,-2 \rangle$.

(a) $x = 2+t$, $y = 8+2t$, $z = 1+3t$

(b) $x = 2+t$, $y = -8+2t$, $z = 1+3t$

(c) $x = 1+2t$, $y = 2+8t$, $z = 3+t$

(d) $x = 1+2t$, $y = 2-8t$, $z = 3+t$

(e) $x = 2-t$, $y = -8-2t$, $z = 1-3t$

$r_0 = (1, 2, 3)$

$v = \langle 3, -1, 2 \rangle \times \langle 1, 0, -2 \rangle$

$= \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = \langle 2, 8, 1 \rangle$

$x = 1+2t,\ y = 2+8t,\ z = 3+t$

10. What is the intersection of the sphere $x^2 + y^2 + z^2 + 6x - 4y - 10z - 26 = 0$ with the yz plane?

(a) The yz plane intersects the sphere in the circle $(y-2)^2 + (z-5)^2 = 55$

(b) The yz plane intersects the sphere at the point $(-3, 2, 5)$

(c) The yz plane intersects the sphere in the circle $(y-2)^2 + (z-5)^2 = 73$

(d) The yz plane does not intersect the sphere.

(e) The yz plane intersects the sphere in the circle $(y-2)^2 + (z-5)^2 = 26$

$y^2 + z^2 - 4y - 10z = 26$

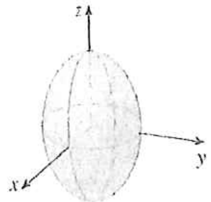
$y^2 - 4y + 4 + z^2 - 10z + 25 =$

$26 + 29$

$(y-2)^2 + (z-5)^2 = 55$

Part II: Work out. Show all intermediate steps.

11. (10 pts) Match the graph with its equation by putting the corresponding letter in the blank provided.



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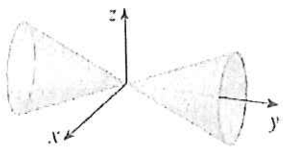

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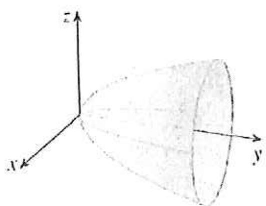

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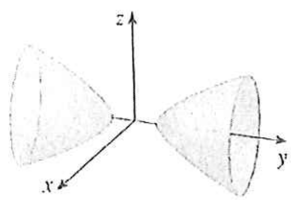

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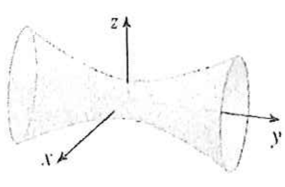

A

A. $\frac{x^2}{4} - \frac{y^2}{9} + \frac{z^2}{9} = 1$

B. $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} = 1$

C. $y = \frac{x^2}{4} + \frac{z^2}{9}$

D. $-\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{9} = 1$

E. $\frac{y^2}{4} = \frac{x^2}{9} + \frac{z^2}{9}$

12. (10 pts) Given that the vector functions $\mathbf{r}_1(t) = \langle -2t, t^5, -5t^3 \rangle$ and $\mathbf{r}_2(u) = \langle \sin(-2u), \sin(u), u - \pi \rangle$ intersect at the origin, find the angle of intersection, rounded to the nearest degree.

$r_1'(t) = \langle -2, 5t^4, -15t^2 \rangle$

$t = 0$

$u = \pi$

$r_2'(u) = \langle -2\cos(2u), \cos u, 1 \rangle$

$r_1'(0) = \langle -2, 0, 0 \rangle$

$r_2'(\pi) = \langle -2, -1, 1 \rangle$

$\cos\theta = \dfrac{\langle -2,0,0\rangle \cdot \langle -2,-1,1\rangle}{2\sqrt{6}}$

$\cos\theta = \dfrac{4}{2\sqrt{6}}$, so $\theta \approx 35^\circ$

13. Consider the planes $x - 3y + z = 11$ and $x - 2y - z = 4$.

a.) (4 pts) Find the point where the line of intersection of these two planes passes through the xy-plane.

set $z = 0$: $x - 3y = 11 \rightarrow x = 11 + 3y$

$x - 2y = 4 \rightarrow 11 + 3y - 2y = 4$

$y = -7$, so $x = 11 - 21$

$x = -10$

$\boxed{(-10, -7, 0)}$

b.) (4 pts) Find a vector, $\mathbf{v}$, that is parallel to the line of intersection of these two planes.

$\vec{v} = \langle 1, -3, 1 \rangle \times \langle 1, -2, -1 \rangle$

$= \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 1 & -2 & -1 \end{vmatrix}$

$\boxed{v = \langle 5, 2, 1 \rangle}$

c.) (6 pts) Using the information in parts a.) and b.), find a parametric equation for the line of intersection of these planes.

$\vec{r}_0 + t\vec{v} = \langle -10, -7, 0 \rangle + t\langle 5, 2, 1 \rangle$

$$\boxed{\begin{aligned} x &= -10 + 5t \\ y &= -7 + 2t \\ z &= t \end{aligned}}$$

14. (8 pts) Find parametric equations for the tangent line to the curve $x = t^2 + t$, $y = 4\sqrt{t}$, $z = e^{t^3 - t}$ at $(2, 4, 1)$.

$r(t) = \langle t^2 + t, 4\sqrt{t}, e^{t^3 - t} \rangle \quad t = 1$

$r'(t) = \langle 2t + 1, \frac{2}{\sqrt{t}}, (3t^2 - 1)e^{t^3 - t} \rangle$

$r'(1) = \langle 3, 2, 2 \rangle$

$\vec{r_0} + t\vec{v} = \langle 2, 4, 1 \rangle + t\langle 3, 2, 2 \rangle$

$x = 2 + 3t$
$y = 4 + 2t$
$z = 1 + 2t$

15. (8 pts) Find the length of the curve $\mathbf{r}(t) = \langle t\sqrt{2}, e^t, e^{-t} \rangle$ for $0 \le t \le 3$.

$L = \int_0^3 |r'(t)|\,dt \qquad r'(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle$

$|r'(t)| = \sqrt{2 + e^{2t} + e^{-2t}}$

$= \int_0^3 (e^t + e^{-t})\,dt$

$= \sqrt{(e^t + e^{-t})^2}$

$= e^t - e^{-t} \Big|_0^3 \qquad = e^t + e^{-t}$

$= \boxed{\cancel{e^3 + e^{-3}}}$

$= \boxed{e^3 - e^{-3}}$

16. (10 pts) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \langle t, e^t, 2t \ln t \rangle$ and $\mathbf{r}(1) = \langle 2, -3, 1 \rangle$. In order to get full credit, steps must be shown if a technique of integration is needed to find the antiderivative.

$$\vec{r}'(t) = \langle t, e^t, 2t \ln t \rangle$$

$$r(t) = \int \langle t, e^t, 2t \ln t \rangle dt$$

$$\int 2t \ln t \, dt \qquad u = \ln t \quad dv = 2t\,dt$$

$$du = \frac{dt}{t} \quad v = t^2$$

$$uv - \int v\,du = t^2 \ln t - \int t\,dt$$

$$= t^2 \ln t - \frac{t^2}{2}$$

$$r(t) = \langle \frac{t^2}{2} + c_1, e^t + c_2, t^2 \ln t - \frac{1}{2}t^2 + c_3 \rangle$$

$$r(1) = \langle \frac{1}{2} + c_1, e + c_2, -\frac{1}{2} + c_3 \rangle = \langle 2, -3, 1 \rangle$$

$$\frac{1}{2} + c_1 = 2 \qquad c_1 = \frac{3}{2}$$

$$e + c_2 = -3 \qquad c_2 = -3 - e$$

$$-\frac{1}{2} + c_3 = 1 \qquad c_3 = \frac{3}{2}$$

3

$$r(t) = \langle \frac{t^2}{2} + \frac{3}{2}, e^t - 3 - e, t^2 \ln t - \frac{1}{2}t^2 + \frac{3}{2} \rangle$$

1 1 5