MATH 251 Spring 2017
EXAM II - VERSION A

LAST NAME: ______________________ FIRST NAME: ______________________

SECTION NUMBER: ____________

UIN: ______________________

DIRECTIONS:

1. This is a non calculator exam. If you are seen using a calculator, your exam will be collected and you will receive a zero on the exam and will be reported to the Honor Council.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 11-17), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the **quality** and **correctness** of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

DO NOT WRITE BELOW!

Question Type	Points Awarded	Points
Multiple Choice		40
Free Response		60
Total		100

PART I: Multiple Choice. 4 points each.

1. Which of the following statements is true regarding $f(x, y) = x^2 + y^2 - 2x + 4y + 2$?
   (a) $f(x, y)$ has a local maximum at $(1, -2, -3)$
   (b) $f(x, y)$ has a local minimum at $(1, -2, -3)$
   (c) $f(x, y)$ has a saddle point at $(1, -2, -3)$
   (d) $f(x, y)$ has no local extrema
   (e) The second derivative test for local extrema fails since $D < 0$

2. If $w = \ln(x + 2y^2 + 3)$, find $w_{xy}(3, 1)$.
   (a) $-\frac{1}{16}$
   (b) $\frac{1}{8}$
   (c) $-\frac{1}{64}$
   (d) $\frac{1}{4}$
   (e) $-\frac{1}{4}$

3. Find the absolute extrema of $f(x, y) = 1 + 4x - 5y$ on the set D, where D is the triangular region with vertices $(0, 0)$, $(0, 3)$ and $(2, 0)$.
   (a) The absolute maximum is $z = 32$, the absolute minimum is $z = -14$.
   (b) The absolute maximum is $z = 9$, the absolute minimum is $z = -14$.
   (c) The absolute maximum is $z = 1$, the absolute minimum is $z = 1$.
   (d) The absolute maximum is $z = 64$, the absolute minimum is $z = -14$.
   (e) The absolute maximum is $z = 32$, the absolute minimum is $z = 1$.

4. Suppose we have a rectangle with length $l = 8$ cm and width $w = 6$ cm. Given there is a maximum error in measurement of 0.2 cm in the length, and a maximum error in measurement of 0.1 cm in the width, use differentials to estimate the maximum error in the calculated area of the rectangle.

   (a) $dA = 2$ square centimeters

   (b) $dA = 0.96$ square centimeters

   (c) $dA = 2.2$ square centimeters

   (d) $dA = 2.8$ square centimeters

   (e) $dA = 1.4$ square centimeters

5. The level curves to the surface $f(x, y) = \dfrac{2x}{x^2 + y^2}$ are

   (a) A family of hyperbolas

   (b) A family of cones

   (c) A family of cylinders

   (d) A family of parabolas

   (e) A family of circles

6. Suppose $u = (r^2 + 2s)^3$, $r = x + y \sin t$, $s = y + x \sin t$. Find $\dfrac{\partial u}{\partial x}$ when $x = 2$, $y = -3$, and $t = 0$.

   (a) $\dfrac{\partial u}{\partial x} = 48$

   (b) $\dfrac{\partial u}{\partial x} = -24$

   (c) $\dfrac{\partial u}{\partial x} = -60$

   (d) $\dfrac{\partial u}{\partial x} = -240$

   (e) $\dfrac{\partial u}{\partial x} = 120$

7. If $f(x, y) = y^2 \tan x$, find the directional derivative at the point $\left(\frac{\pi}{4}, -3\right)$ in the direction of $\theta = -\frac{\pi}{4}$

(a) $12\sqrt{2}$

(b) $15\sqrt{2}$

(c) $18\sqrt{2}$

(d) $\frac{9}{2}\sqrt{2} + 3\sqrt{2}$

(e) $\frac{9}{2}\sqrt{2} - 3\sqrt{2}$

8. Find the equation of the tangent plane to the surface $z = e^{x^3+y^5}$ at the point $(1, -1, 1)$.

(a) $3x + 5y - z = -7$

(b) $3x - 5y - z = -9$

(c) $-3x - 5y - z = 3$

(d) $3x + 5y - z = -3$

(e) $-x - y + z = 1$

9. Find the directional derivative of $f(x, y) = x^2 - y^2 + 2xy$ at the point $P(2, -3)$ in the direction $\mathbf{i} - \mathbf{j}$.

(a) -12

(b) $\frac{8}{\sqrt{2}}$

(c) 10

(d) $-\frac{12}{\sqrt{2}}$

(e) $-\frac{10}{\sqrt{2}}$

10. For any given surface $z = f(x, y)$, what is the geometric interpretation of $f_x(1, 2)$?

(a) The slope of the curve at the point $(1, 2, f(1, 2))$ obtained by intersecting the surface with the plane $y = 2$.

(b) The slope of the curve at the point $(1, 2, f(1, 2))$ obtained by intersecting the surface with the plane $x = 1$.

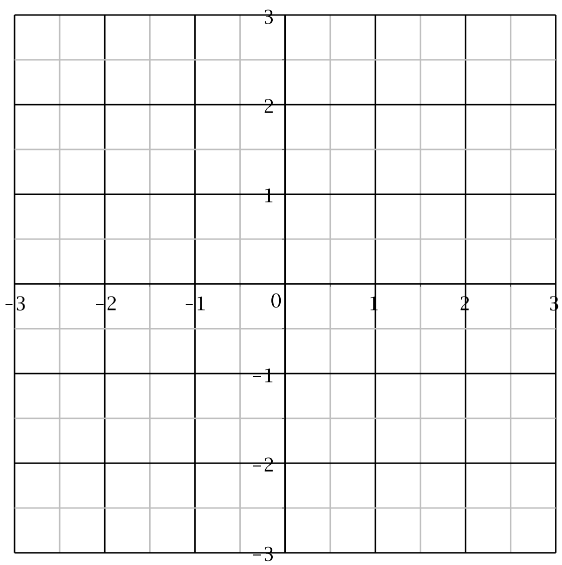
(c) The directional derivative of $f(x, y)$ at the point $(1, 2, f(1, 2))$ in the direction of $\mathbf{i}$.

(d) Both (a) and (c) are true.

(e) Both (b) and (c) are true.

Part II: Work out. Show all intermediate steps

11. (8 pts) Sketch the domain of $f(x, y) = \ln(4 - x^2 - y^2) + \sqrt{xy}$.



12. (6 pts) If If $f(x,y) = e^{xy} + ye^x$, find the maximum rate of change at the point $P(-1,1)$ and the direction in which it occurs. *Put your final answer in the blanks provided below*

Maximum rate of change:____________________ **Direction:**____________________

13. (10 pts) Find all local extrema/saddle points for $f(x,y) = x^2 + y^2 + x^2y + 4$. *Put your final answer in the blanks provided below.* If there are none, write NONE in the answer space. If there is more than one, separate by a comma.

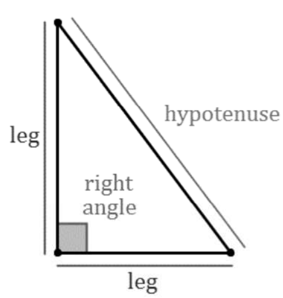
Max:__________________ **Min:**__________________ **SP:**__________________

14. Consider the surface $z + e = xe^y \cos z$, find:

a.) (8 pts) The equation of the tangent plane to the surface at the point $(1, 1, 0)$.

b.) (5 pts) The equation of the normal line to the surface at the point $(1, 1, 0)$.

15. (7 pts) At a certain instant, the legs of a right triangle have lengths 2 and 4 feet, and they are increasing at the rates of 1 foot per minute and 2 feet per minute, respectively. How fast is the area of the triangle changing at that moment? Be sure to label all variables.



16. (6 pts) Suppose we have 64 square feet of cardboard in order to make a closed rectangular box with length l, width w, and height h. Find a formula for the volume of the box **in terms of** l **and** w **only** that we would use in order to maximize the volume of this box. **DO NOT SOLVE THE PROBLEM**!!!! *Put your answer in the blank provided below.*

$V(l, w) =$ ____

17. (10 pts) Find the absolute extrema for $f(x, y) = 2x^3 + y^2 + 3$ on the set $D = \{(x, y)|x^2 + y^2 \leq 1\}$. Show all work. For the final answer, only the z coordinate is required, simplified. *Put your final answer in the blanks provided below.*

Absoute Maximum : $z =$ _______ **Absoute Minimum :** $z =$ _______

18. THIS PROBLEM WAS ADDED FOR CONTENT. Use Lagrange Multipliers to find the absolute maximum value of $f(x, y, z) = xyz$ subject to the constraint $2x + y + 3z = 60$.

19. THIS PROBLEM WAS ADDED FOR CONTENT. If $f(x, y) = 2xy - 3y - 2x + 5$, find the absolute maximum and minimum over the closed triangular region with vertices $(0, 0)$, $(2, 0)$ and $(2, 4)$. Put ALL consideration points in the table provided.

critical points	function value