Questions asked in class

① How do you know what formula to use for the equation of a tangent plane?

I $z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

use if $z = f(x, y)$

Find tangent plane to $z = x^2 y^3$ at $(2,1)$

$x_0 = 2$ $\quad f_x = 2xy^3$ $\quad f_x(2,1) = 4$

$y_0 = 1$ $\quad f_y = 3x^2y^2$ $\quad f_y(2,1) = 12$

$z_0 = 4$

$$\boxed{z - 4 = 4(x-2) + 12(y-1)}$$

II Formula to use if the surface is not explictly solved for z.

Find tangent plane to the surface

$2xy + 3yz + 7xz = -9$

at $(1, 2, -1)$

$\vec{n} \cdot (\vec{r} - \vec{r_0}) = 0$ $\quad \vec{n} = \nabla F$, where

$F(x, y, z) = 2xy + 3yz + 7xz$

$\nabla F = \langle F_x, F_y, F_z \rangle$

$\nabla F = \langle 2y + 7z, 2x + 3z, 3y + 7x \rangle$

$\vec{n} = \nabla F(1, 2, -1) = \langle 4 - 7, 2 - 3, 6 + 7 \rangle$

$\vec{n} = \langle -3, -1, 13 \rangle$

$n \cdot (r - r_0) = 0 \to \langle -3, -1, 13 \rangle \cdot \langle x - 1, y - 2, z + 1 \rangle = 0$

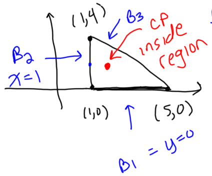
$$\boxed{-3(x-1) - (y-2) + 13(z+1) = 0}$$

Find the absolute maximum and minimum values of f on the set D.

$f(x, y) = 8 + xy - x - 2y$, D is the closed triangular region with vertices (1, 0), (5, 0), and (1, 4)

absolute maximum value ✗ 7

absolute minimum value ✗ 3



$f_y = 0 \rightarrow x - 2 = 0 \quad x = 2$

$f_x = 0 \rightarrow y - 1 = 0 \quad y = 1$

$\boxed{CP: (2,1)}$

$f(2,1) = 8 + 2 - 2 - 2$

$\boxed{f(2,1) = \boxed{6}}$

Three Boundary curves: B1 : $\boxed{y=0}$

B2 : $\boxed{x=1}$

B3 : $y = 5 - x$

$f(x,y) = 8 + xy - x - 2y$

B1 : $f(x,0) = 8 - x, \quad 1 \le x \le 5$

cn: none! $\quad f(1,0) = \boxed{7}$

$f(5,0) = 8 - 5 = \boxed{3}$

B2: $f(1,y) = 8 + y - 1 - 2y, \quad 0 \le y \le 4$

$= 7 - y \quad f(1,0) = \boxed{7}$

cn: none! $\quad f(1,4) = 7 - 4 = \boxed{3}$

B3: $y = 5 - x$:

$f(x, 5-x) = 8 + x(5-x) - x - 2(5-x), \quad 1 \le x \le 5$

$= 8 + 5x - x^2 - x - 10 + 2x$

$= -x^2 + 6x - 2, \quad 1 \le x \le \boxed{5}$

$f'(x, 5-x) = -2x + 6 \quad$ cn: $x = 3$

$x = 3$: $f(3,2) = -9 + 18 - 2 = \boxed{7}$

$x = 1$: $f(1,4) = -1 + 6 - 2 = \boxed{3}$

$x = 4$: $f(5,0) = -25 + 30 - 2 = \boxed{3}$

ABS max = 7

ABS mn = 3