MATH 151
FALL 2009

SAMPLE EXAM III

Part I - Multiple Choice

1. Given $f(x) = x^3 \ln x$, find $f'(e)$

a) e b) $3 + 3e^2$ c) e^2

d) $3e$ e) $4e^2$

2. $\dfrac{d}{dx}(\tan^{-1}(x^2)) =$

a) $\dfrac{2x}{1+x^4}$ b) $\dfrac{2}{1+x^2}$ c) $-2x\csc(x^2)\cot(x^2)$

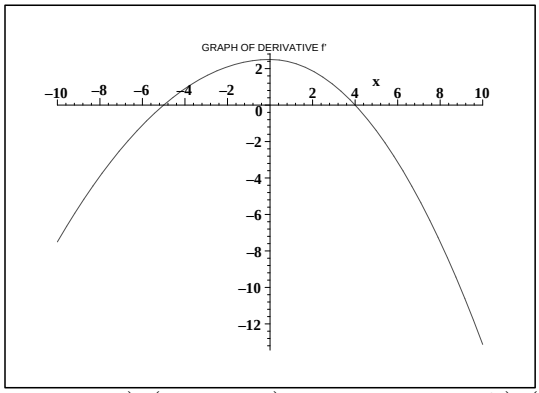
d) $2x\tan^{-1}(x^2)\sec^{-1}(x^2)$ e) $\dfrac{2x}{1+x^2}$

3. Find the value of c that satisfies the Mean Value Theorem for $f(x) = x^2 + 4$ on the interval $[-1, 2]$.

a) $\dfrac{7}{4}$ b) $\dfrac{2}{3}$ c) $\dfrac{1}{3}$

d) 1 e) $\dfrac{1}{2}$

4. The graph of the *DERIVATIVE* of a function is shown below. On which intervals is the original function f concave down?
CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!



a) $(-\infty, -5)$ b) $(-5, 0)$

c) $(0, 4)$ d) $(4, \infty)$

e) none of these intervals

5. Circle ALL the critical values of $f(x) = x(x-1)^{\frac{1}{3}}$
NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!

a) 0 b) $-\frac{1}{3}$ c) $\frac{1}{4}$

d) $\frac{3}{4}$ e) 1

6. Find the absolute maximum of $f(x) = \sin x + \cos x$ on the interval $\left[0, \frac{\pi}{3}\right]$.
(**NOTE**: $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.73$)

a) 1 b) 2 c) $\sqrt{2}$

d) $\frac{\sqrt{3}+1}{2}$ e) $\frac{\pi}{4}$

7. The inflection points of $f(x) = x^5 + 10x^4$ occur at which of the following?

a) $x = 6$ only b) $x = -6$ only

c) $x = 0$, $x = -8$ d) $x = 0$, $x = -6$

e) $x = 0$, $x = 6$

8. Which is an antiderivative of $f(x) = 2\sqrt{x} + \frac{1}{x^2}$?

a) $\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{x} + C$ b) $\frac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C$

c) $\frac{1}{\sqrt{x}} - \frac{2}{x^3} + C$ d) $\frac{1}{\sqrt{x}} + \ln(x^2) + C$

e) $\frac{1}{\sqrt{x}} - \frac{1}{x} + C$

9. Write $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$ in summation notation.

a.) $\sum_{n=1}^{6} \frac{(-1)^{n+1}}{2^n}$

b.) $\sum_{n=1}^{6} \frac{(-1)^{n}}{2^n}$

c.) $\sum_{n=1}^{6} \frac{(-1)^{n+1}}{2n}$

d.) $\sum_{n=1}^{6} \frac{(-1)^{n}}{2n}$

e.) $\sum_{n=1}^{6} \frac{-1}{2^n}$

10. Approximate the area under the graph of $f(x) = \ln x$, above the x-axis on the interval $[1, 3]$ using 4 subintervals of equal width and midpoints.

a.) $\ln \dfrac{3465}{256}$

b.) $\dfrac{1}{2} \ln \dfrac{3465}{256}$

c.) $\dfrac{1}{2} \ln \dfrac{32}{16}$

d.) $\ln \dfrac{32}{16}$

e.) None of the above.

11. Find $\lim\limits_{x \to 0} \dfrac{\arctan x - x}{x^3}$

a.) $-\dfrac{1}{3}$

b.) $-\dfrac{1}{2}$

c.) 0

d.) ∞

e.) -6

Part II - Work Out Problems

12. Find the derivative of $f(x) = x^{\sec x} + 2^{\arcsin x}$

13. Find $\lim_{x \to 0}(1-x)^{\frac{5}{x}}$

14. On what intervals is $f(x) = x + 2\cos x$, $0 \leq x \leq 2\pi$, increasing?

15. Find the intervals of concavity for $f(x) = xe^{4x}$

16. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 2.

17. The acceleration of a particle is given by $\mathbf{a}(t) = (1+e^t)\mathbf{i} + (\cos t)\mathbf{j}$. If the initial velocity is $\mathbf{i}$ and the initial position is $\mathbf{j}$, find the position of the particle at any time t.

18. A thermometer is taken from a room where the temperature is 75° to the outdoors, where the temperature is 35°. After one minute, the thermometer reads 60°. What is the reading of the thermometer at time t?

19. By introducing a deadly chemical, the population of a particular bacteria culture is decreasing over time. Suppose it has been determined that the rate of change of the population at time t minutes is $\frac{1}{3}$ of the population. If the initial size of the population is 1000 bacteria, how many bacteria are present after 3 minutes?

20. Find the following:

a.) $\arccos(\frac{1}{2}) =$ ____

b.) $\sin(\arccos(-\frac{4}{5})) =$ ____

c.) $\arcsin(\sin(\frac{5\pi}{6})) =$ ____

d.) The domain of $\arccos(4x-5) =$ _____

e.) $\cos(\arctan x) =$ _____