

MATH 151
FALL 2009

SAMPLE EXAM III

Part I - Multiple Choice

1. Given $f(x) = x^3 \ln x$, find $f'(e)$

- a) e b) $3 + 3e^2$ c) e^2
d) $3e$ e) $4e^2$

2. $\frac{d}{dx}(\tan^{-1}(x^2)) =$

- a) $\frac{2x}{1+x^4}$ b) $\frac{2}{1+x^2}$ c) $-2x \csc(x^2) \cot(x^2)$
d) $2x \tan^{-1}(x^2) \sec^{-1}(x^2)$ e) $\frac{2x}{1+x^2}$

3. Find the value of c that satisfies the Mean Value Theorem for $f(x) = x^2 + 4$ on the interval $[-1, 2]$.

a) $\frac{7}{4}$

b) $\frac{2}{3}$

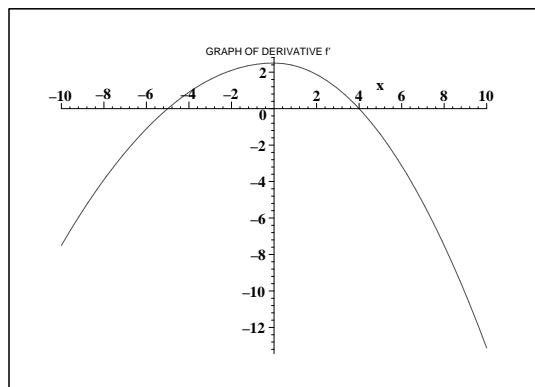
c) $\frac{1}{3}$

d) 1

e) $\frac{1}{2}$

4. The graph of the *DERIVATIVE* of a function is shown below. On which intervals is the original function f concave down?

CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!



a) $(-\infty, -5)$

b) $(-5, 0)$

c) $(0, 4)$

d) $(4, \infty)$

e) none of these intervals

5. Circle ALL the critical values of $f(x) = x(x - 1)^{\frac{1}{3}}$

NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!

a) 0

b) $-\frac{1}{3}$

c) $\frac{1}{4}$

d) $\frac{3}{4}$

e) 1

6. Find the absolute maximum of $f(x) = \sin x + \cos x$ on the interval $\left[0, \frac{\pi}{3}\right]$.
(NOTE: $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.73$)

a) 1 b) 2 c) $\sqrt{2}$

d) $\frac{\sqrt{3} + 1}{2}$ e) $\frac{\pi}{4}$

7. The inflection points of $f(x) = x^5 + 10x^4$ occur at which of the following?

- a) $x = 6$ only b) $x = -6$ only
c) $x = 0, x = -8$ d) $x = 0, x = -6$
e) $x = 0, x = 6$

8. Which is an antiderivative of $f(x) = 2\sqrt{x} + \frac{1}{x^2}$?

a) $\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{x} + C$

b) $\frac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C$

c) $\frac{1}{\sqrt{x}} - \frac{2}{x^3} + C$

d) $\frac{1}{\sqrt{x}} + \ln(x^2) + C$

e) $\frac{1}{\sqrt{x}} - \frac{1}{x} + C$

9. Write $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \frac{1}{64}$ in summation notation.

a.) $\sum_{n=1}^6 \frac{(-1)^{n+1}}{2^n}$

b.) $\sum_{n=1}^6 \frac{(-1)^n}{2^n}$

c.) $\sum_{n=1}^6 \frac{(-1)^{n+1}}{2n}$

d.) $\sum_{n=1}^6 \frac{(-1)^n}{2n}$

e.) $\sum_{n=1}^6 \frac{-1}{2^n}$

10. Approximate the area under the graph of $f(x) = \ln x$, above the x -axis on the interval $[1, 3]$ using 4 subintervals of equal width and midpoints.

a.) $\ln \frac{3465}{256}$

b.) $\frac{1}{2} \ln \frac{3465}{256}$

c.) $\frac{1}{2} \ln \frac{32}{16}$

d.) $\ln \frac{32}{16}$

e.) None of the above.

11. Find $\lim_{x \rightarrow 0} \frac{\arctan x - x}{x^3}$

a.) $-\frac{1}{3}$

b.) $-\frac{1}{2}$

c.) 0

d.) ∞

e.) -6

Part II - Work Out Problems

12. Find the derivative of $f(x) = x^{\sec x} + 2^{\arcsin x}$

13. Find $\lim_{x \rightarrow 0} (1 - x)^{\frac{5}{x}}$

14. On what intervals is $f(x) = x + 2 \cos x$, $0 \leq x \leq 2\pi$, increasing?

15. Find the intervals of concavity for $f(x) = xe^{4x}$

16. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 2.

17. The acceleration of a particle is given by $\mathbf{a}(t) = (1 + e^t)\mathbf{i} + (\cos t)\mathbf{j}$. If the initial velocity is \mathbf{i} and the initial position is \mathbf{j} , find the position of the particle at any time t .

18. A thermometer is taken from a room where the temperature is 75° to the outdoors, where the temperature is 35° . After one minute, the thermometer reads 60° . What is the reading of the thermometer at time t ?

19. By introducing a deadly chemical, the population of a particular bacteria culture is decreasing over time. Suppose it has been determined that the rate of change of the population at time t minutes is $\frac{1}{3}$ of the population. If the initial size of the population is 1000 bacteria, how many bacteria are present after 3 minutes?

20. Find the following:

a.) $\arccos\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$

b.) $\sin\left(\arccos\left(-\frac{4}{5}\right)\right) = \underline{\hspace{2cm}}$

c.) $\arcsin\left(\sin\left(\frac{5\pi}{6}\right)\right) = \underline{\hspace{2cm}}$

d.) The domain of $\arccos(4x - 5) = \underline{\hspace{4cm}}$

e.) $\cos(\arctan x) = \underline{\hspace{4cm}}$