Section 10.1: Sequences

Def: A sequence is an ordered list of numbers a_1, a_2, a_3,...,a_{100},... Notation often used is $\{a_n\}_{n=1}^{\infty} = a_1, a_2, a_3, ..., a_{100}, ..$,or just abbreviated as $\{a_n\} = a_1, a_2, a_3, ..., a_{100}, ..$ since the starting point of a sequence is not important when finding the limit of a sequence. We want to know what happens to the terms of a sequence for *LARGE* values of n.

1. Write out the first 5 terms of the sequence $\{\frac{n+1}{n+3}\}_{n=1}^{\infty}$.

2. Find a general formula for the sequence:

   a.) $\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, ...$

   b.) $-\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, ...$

Def: If $\lim_{n\to\infty} a_n = L$, then we say the sequence $\{a_n\}$ *CONVERGES* to L. If $\lim_{n\to\infty} a_n = \infty$ or does not exist, then we say the sequence $\{a_n\}$ *DIVERGES*.

3. Determine whether the following sequences converge or diverge. If the sequence converges, find the limit. If the sequence diverges, explain why.

   a.) $a_n = \sqrt{\dfrac{4n+3}{7n+6}}$

   b.) $a_n = \arccos \dfrac{n+1}{2n+3}$

   c.) $a_n = \ln(3n+1) - \ln(4n)$

d.) $a_n = \dfrac{\cos n}{n^3}$

e.) $a_n = \dfrac{(-1)^n n}{n^3 + 1}$

f.) $a_n = \dfrac{(-1)^n n}{3n + 1}$

g.) $a_n = \cos \dfrac{n\pi}{2}$

Def: We say a sequence is bounded below if there is a number N so that $a_n \geq N$ for all n. We say a sequence is bounded above if there is a number M so that $a_n \leq M$ for all n. If a_n is bounded both above and below, then we say the sequence is bounded.

4. Determine whether the sequence is bounded:

   a.) $a_n = \left\{ \dfrac{1}{n^2} \right\}_{n=1}^{\infty}$

   b.) $a_n = \left\{ \dfrac{n^2}{n+1} \right\}_{n=1}^{\infty}$

Def: We say a sequence a_n is increasing if $a_n < a_{n+1}$ for all $n \geq 1$. We say a sequence a_n is decreasing if $a_n > a_{n+1}$ for all $n \geq 1$. Note: For a sequence to be increasing or decreasing, it need not be true for ALL n, it just must be eventually true, from some value m to ∞. If a sequence is either increasing or decreasing, then we say the sequence is monotonic.

5. Determine whether following sequences are increasing, decreasing, or not monotonic.

   a.) $a_n = \dfrac{3}{n+5}$

b.) $a_n = \dfrac{n^2 + 4n + 3}{n^2}$

c.) $a_n = \cos \dfrac{n\pi}{2}$

6. Consider the sequence defined by $a_1 = 2$, $a_{n+1} = \dfrac{1}{3 - a_n}$. Find the first 5 terms of the sequence. Find the limit of the sequence.

7. For what values of r does the sequence $\{r^n\}$ converge? For these values of r, find $\lim_{n\to\infty} r^n$.