Section 10.5: Power series

Def: A **Power Series** is a series of the form $\sum\limits_{n=1}^{\infty} c_n(x-a)^n$, where x is the variable and the c_n's are called the coefficients of the series. More generally, $\sum\limits_{n=1}^{\infty} c_n(x-a)^n$ is called a power series *centered* at $x = a$, or a power series *about* a. Specifically, $\sum\limits_{n=1}^{\infty} c_n x^n$ is a power series centered at zero.

Theorem: For a given power series $\sum\limits_{n=1}^{\infty} c_n(x-a)^n$ there are only three possibilities:

(i) The series converges only for $x = a$.

(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$. Here, we call R the radius of convergence.

EXAMPLE 1: For the following power series, find the radius and interval of convergence.

a.) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n^2 + 1}$

b.) $\sum_{n=1}^{\infty} \frac{(-3)^n(2x-1)^n}{n}$

c.) $\sum_{n=0}^{\infty} \frac{(2n)!(x-5)^n}{7^{n-1}}$

d.) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$

e.) $\sum_{n=2}^{\infty} \frac{(-1)^n (x-1)^n}{2^n \ln n}$

EXAMPLE 2: Suppose it is known that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series:

a.) $\sum_{n=0}^{\infty} c_n (2)^n$

b.) $\sum_{n=0}^{\infty} c_n (8)^n$

c.) $\sum_{n=0}^{\infty} c_n (4)^n$

d.) $\sum_{n=0}^{\infty} c_n (-5)^n$