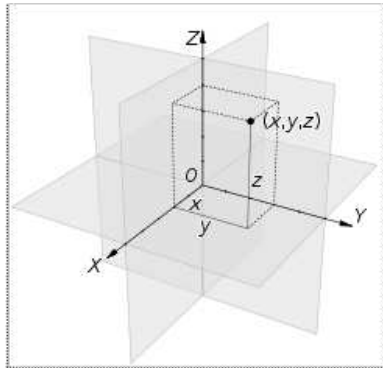


## Section 11.1: Three-Dimensional Coordinate Systems

In order to represent points in space, we first choose a fixed point  $O$  (the origin) and three directed lines through  $O$  perpendicular to each other, called the **coordinate axes**, and labeled the  $x$ -axis,  $y$ -axis and  $z$ -axis. We call the  $xy$ ,  $xz$  and  $yz$  planes the **coordinate planes**.



*EXAMPLE 1:* Plot the points  $P(1, 5, 3)$  and  $Q(-2, 1, 2)$  on the same coordinate axis. What is the projection of  $P$  onto the three coordinate planes?

*EXAMPLE 2:* Describe the planes  $z = 4$  and  $y = 3$  in  $R^3$ .

Distance formula in three dimensions: The distance from the points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  is  $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ .

*EXAMPLE 3:* Find the distance between the points in example 1:  $P(1, 5, 3)$  and  $Q(-2, 1, 2)$ .

*EXAMPLE 4:* Given the points  $A(5, 5, 1)$ ,  $B(3, 3, 2)$  and  $C(1, 4, 4)$ , determine whether  $\triangle ABC$  is isosceles, right, or both.

*EXAMPLE 5:* Find the equation of the sphere with radius 7 and center  $(-1, 2, 7)$ . What is the intersection of this sphere with the  $xz$  plane? With the  $xy$  plane?

*EXAMPLE 6:* Find the center and radius of the sphere  $x^2 + y^2 + z^2 = 6x - 4y + 10z$ .

*EXAMPLE 7:* Find the sphere with center  $(2, -3, 6)$  that touches the  $xy$  plane.

*EXAMPLE 8:* Describe the following regions in the dimension indicated.

a.)  $x = 4$  in  $R^2$  and  $R^3$

b.)  $x^2 + y^2 \leq 4$  in  $R^2$  and  $R^3$

c.)  $xz = 0$  in  $R^3$

d.)  $z \leq 0$  in  $R^3$

e.)  $1 \leq x^2 + y^2 + z^2 \leq 25$  in  $R^3$