Section 11.2: Vectors

A three dimensional vector is an ordered triple $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$. The numbers a_1, a_2 and a_3 are called the **components** of the vector $\overrightarrow{a}$. If a vector starts at the point $A(a_1, a_2, a_3)$ and ends at the point $B(b_1, b_2, b_3)$, then the vector with representation $\overrightarrow{AB}$ is given by $\overrightarrow{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3 \rangle$. The case where the initial point is the origin is called the **position vector**.

EXAMPLE 1: Find the components of the vector with initial point $A(-2, 4, 7)$ and terminal point $B(3, 2, -5)$.

The Algebra of Vectors: Suppose $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$ and $\overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$ are vectors and c is a scalar.

a.) Scalar Multiplication: $c\overrightarrow{a} = \langle ca_1, ca_2, ca_3 \rangle$.

b.) Vector Sum: $\overrightarrow{a+b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$.

c.) Vector Difference: $\overrightarrow{a-b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$.

d.) Vector Length: $|\overrightarrow{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$.

e.) Unit Vector: A unit vector in the direction of $\overrightarrow{a}$ is $\overrightarrow{u} = \dfrac{\overrightarrow{a}}{|\overrightarrow{a}|}$

f.) Dot Product: $\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}| \cos\theta$, where θ is the angle between the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$.

Illustration:

EXAMPLE 2: Given $\overrightarrow{a} = \langle 1, 2, -1 \rangle$ and $\overrightarrow{b} = \langle 0, 3, -5 \rangle$, find:

a.) $\overrightarrow{a + 2b}$

b.) $|\overrightarrow{a - b}|$

c.) A unit vector in the direction of $\overrightarrow{b}$.

EXAMPLE 3: What is the value of $\overrightarrow{a} \cdot \overrightarrow{b}$ if it is known that $\overrightarrow{a}$ and $\overrightarrow{b}$ are perpendicular?

EXAMPLE 4: Find $\overrightarrow{a} \cdot \overrightarrow{b}$ if it is known that $|\overrightarrow{a}| = 2$, $|\overrightarrow{b}| = 5$ and the angle between the vectors is $60°$.

Alternative Dot Product: $\overrightarrow{a} \cdot \overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3$.

EXAMPLE 5: Find the dot product between the vectors $\langle 1, 5, -2 \rangle$ and $\langle 3, 1, 4 \rangle$.

EXAMPLE 6: For what value(s) of x are the vectors $\langle x, 1, 2 \rangle$ and $\langle 3, 4, x \rangle$ perpendicular?

EXAMPLE 7: Find the angle between the vectors $\langle 1, 0, -2\rangle$ and $\langle 2, -1, 3\rangle$.

EXAMPLE 8: The points $A(0, -1, 6)$, $B(2, 1, -3)$ and $C(5, 4, 2)$ form a triangle. Find the angles of ΔABC.

Vector and Scalar Projections: Given $\overrightarrow{a} = \langle a_1, a_2, a_3 \rangle$ and $\overrightarrow{b} = \langle b_1, b_2, b_3 \rangle$, we want to project $\overrightarrow{b}$ onto $\overrightarrow{a}$.

- The **Vector Projection** of $\overrightarrow{b}$ onto $\overrightarrow{a}$ is:

$$\overrightarrow{proj_a b} = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|^2} \overrightarrow{a}$$

- The **Scalar Projection** of $\overrightarrow{b}$ onto $\overrightarrow{a}$ (also called the component of $\overrightarrow{b}$ onto $\overrightarrow{a}$) is:

$$comp_a b = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|}$$

EXAMPLE 9: Find the vector and scalar projection of $\langle 1, -1, 3\rangle$ onto $\langle 0, 2, 1\rangle$.