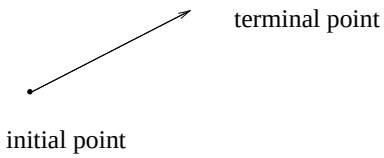
Section 1.1: Vectors

Definition: A **Vector** is a quantity that has both magnitude and direction. Specifically, a two-dimensional vector is an ordered pair $\vec{a} = \langle a_1, a_2 \rangle$. We call a_1 and a_2 the *components* of the vector $\vec{a}$. Moreover, a_1 is how far we move in the x direction to get from the initial point to the terminal point, and a_2 is how far we move in the y direction to get from the initial point to the terminal point.



The special case where the initial point is located at the origin is called the *position vector*.

EXAMPLE 1: Draw the vector with initial point $A(1, 2)$ and terminal point $B(3, -2)$. What are the components of $\vec{AB}$?

EXAMPLE 2: Find the components of the vector $\vec{r}$ given that:

(a) $|\vec{r}| = 2$ and $\vec{r}$ makes an angle of $60°$ with the positive x-axis.

(b) $|\vec{r}| = 7$ and $\vec{r}$ makes an angle of $150°$ with the positive x-axis.

(c) $|\vec{r}| = \dfrac{1}{2}$ and $\vec{r}$ makes an angle of $-45°$ with the positive x-axis.

<u>The Algebra of Vectors</u>: Suppose $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$.

(1) Scalar Multiplication: If c is a scalar and $\mathbf{a} = \langle a_1, a_2 \rangle$ is a vector, then $c\mathbf{a} = \langle ca_1, ca_2 \rangle$.

(2) Vector Sum: $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$.

(3) Vector Difference: $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$.

(4) Vector Length: $|\mathbf{a}| = \sqrt{(a_1)^2 + (a_2)^2}$.

(5) Unit Vector: A **unit vector** is a vector with length one. A unit vector in the direction of $\mathbf{a}$ is $\mathbf{u} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$.

(6) Basis Vectors: Def: $\vec{i} = \langle 1, 0 \rangle$ and $\vec{j} = \langle 0, 1 \rangle$

<u>Illustration:</u>

EXAMPLE 3: Given $\mathbf{a} = \langle -1, 2 \rangle$ and $\mathbf{b} = \langle 4, 3 \rangle$, find:

(a) $3\mathbf{a} + 4\mathbf{b} - \mathbf{i}$

(b) $|\mathbf{a} - \mathbf{b}|$

(c) A unit vector in the direction of $\mathbf{b}$.

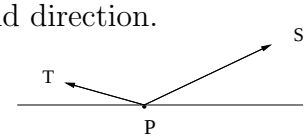
(d) A vector with length 3 in the direction of $\mathbf{b}$.

EXAMPLE 4: Suppose $\mathbf{a} = \langle 1, 5 \rangle$, $\mathbf{b} = \langle 3, -1 \rangle$, and $\mathbf{c} = \langle 8, 6 \rangle$. Find scalars t and w so that $t\mathbf{a} + w\mathbf{b} = \mathbf{c}$.

Applications to Physics and Engineering: A force is represented by a vector because it has both magnitude (measured in pounds or newtons) and direction. If several forces are acting on an object, the **resultant force** experienced by the object is the vector sum of the forces.

EXAMPLE 5: Ben walks due west on the deck of a ship at 3 mph. The ship is moving north at 22 mph. Find the speed and direction of Ben relative to the surface of the water.

EXAMPLE 6: Two forces, $\vec{S}$ and $\vec{T}$, are acting on an object at a point P as shown. $|\vec{S}| = 20$ pounds and measures a reference angle of $45°$. $|\vec{T}| = 16$ pounds and measures a reference angle of $30°$. Find the resultant force as well as its magnitude and direction.



EXAMPLE 7: Suppose that a wind is blowing from the direction $N45°W$ at a speed of 50 km/hr. A pilot is steering a plane in the direction $N60°E$ at an airspeed (speed in still air) of 250 km/hr. Find the true course (direction of the resultant velocity vectors of the plane and wind) and ground speed (magnitude of resultant).

EXAMPLE 8: Two ropes are used to suspend an 80 pound weight as shown. Find the magnitude of the tension in each rope.