Section 1.1: Vectors

**Definition:** A Vector is a quantity that has both magnitude and direction. Specifically, a two-dimensional vector is an ordered pair \( \vec{a} = \langle a_1, a_2 \rangle \). We call \( a_1 \) and \( a_2 \) the *components* of the vector \( \vec{a} \). Moreover, \( a_1 \) is how far we move in the \( x \) direction to get from the initial point to the terminal point, and \( a_2 \) is how far we move in the \( y \) direction to get from the initial point to the terminal point.

The special case where the initial point is located at the origin is called the *position vector*.  

**EXAMPLE 1:** Draw the vector with initial point \( A(1, 2) \) and terminal point \( B(3, -2) \). What are the components of \( \vec{AB} \)?

**EXAMPLE 2:** Find the components of the vector \( \vec{r} \) given that:

(a) \( |\vec{r}| = 2 \) and \( \vec{r} \) makes an angle of \( 60^\circ \) with the positive \( x \)-axis.

(b) \( |\vec{r}| = 7 \) and \( \vec{r} \) makes an angle of \( 150^\circ \) with the positive \( x \)-axis.

(c) \( |\vec{r}| = \frac{1}{2} \) and \( \vec{r} \) makes an angle of \( -45^\circ \) with the positive \( x \)-axis.
The Algebra of Vectors: Suppose \( \mathbf{a} = \langle a_1, a_2 \rangle \) and \( \mathbf{b} = \langle b_1, b_2 \rangle \).

(1) Scalar Multiplication: If \( c \) is a scalar and \( \mathbf{a} = \langle a_1, a_2 \rangle \) is a vector, then \( \mathbf{ca} = \langle ca_1, ca_2 \rangle \).

(2) Vector Sum: \( \mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \).

(3) Vector Difference: \( \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle \).

(4) Vector Length: \( |\mathbf{a}| = \sqrt{(a_1)^2 + (a_2)^2} \).

(5) Unit Vector: A unit vector is a vector with length one. A unit vector in the direction of \( \mathbf{a} \) is \( \mathbf{u} = \frac{\mathbf{a}}{|\mathbf{a}|} \).

(6) Basis Vectors: Def: \( \mathbf{i} = \langle 1, 0 \rangle \) and \( \mathbf{j} = \langle 0, 1 \rangle \)

Illustration:
EXAMPLE 3: Given \( \mathbf{a} = \langle -1, 2 \rangle \) and \( \mathbf{b} = \langle 4, 3 \rangle \), find:

(a) \( 3\mathbf{a} + 4\mathbf{b} - \mathbf{i} \)

(b) \( |\mathbf{a} - \mathbf{b}| \)

(c) A unit vector in the direction of \( \mathbf{b} \).

(d) A vector with length 3 in the direction of \( \mathbf{b} \).
EXAMPLE 4: Suppose $\mathbf{a} = \langle 1, 5 \rangle$, $\mathbf{b} = \langle 3, -1 \rangle$, and $\mathbf{c} = \langle 8, 6 \rangle$. Find scalars $t$ and $w$ so that $t\mathbf{a} + w\mathbf{b} = \mathbf{c}$.

Applications to Physics and Engineering: A force is represented by a vector because it has both magnitude (measured in pounds or newtons) and direction. If several forces are acting on an object, the **resultant force** experienced by the object is the vector sum of the forces.

EXAMPLE 5: Ben walks due west on the deck of a ship at 3 mph. The ship is moving north at 22 mph. Find the speed and direction of Ben relative to the surface of the water.
EXAMPLE 6: Two forces, \( \vec{S} \) and \( \vec{T} \), are acting on an object at a point \( P \) as shown. 
\( |\vec{S}| = 20 \) pounds and measures a reference angle of \( 45^\circ \). 
\( |\vec{T}| = 16 \) pounds and measures a reference angle of \( 30^\circ \). Find the resultant force as well as its magnitude and direction.
EXAMPLE 7: Suppose that a wind is blowing from the direction $N45^\circ W$ at a speed of 50 km/hr. A pilot is steering a plane in the direction $N60^\circ E$ at an airspeed (speed in still air) of 250 km/hr. Find the true course (direction of the resultant velocity vectors of the plane and wind) and ground speed (magnitude of resultant).
EXAMPLE 8: Two ropes are used to suspend an 80 pound weight as shown. Find the magnitude of the tension in each rope.