Section 1.3: Vector Functions and Parametric Curves

Parametric Curves: We call $x = f(t)$ and $y = g(t)$ parametric equations, where t is the parameter. As t varies over its domain, we get a collection of points $(x, y) = (f(t), g(t))$ which traces out the parametric curve.

EXAMPLE 1: Sketch the parametric curves described below:

(a) $x = t - 3$, $y = 2t - 1$

(b) $x = 1 - 2t$, $y = 2 + 3t$, $-3 \leq t < 3$

(c) $x = t + 1$, $y = t^2 - 4$

(d) $x = \sqrt{t}$, $y = 1 - t$

(e) $x = 2\sin\theta$, $y = 3\cos\theta$

(f) $x = \sin t$, $y = \csc t$, $\dfrac{\pi}{6} \leq t < \dfrac{\pi}{2}$

<u>Vector Functions</u>: We call $\vec{r}(t) = \langle x(t), y(t) \rangle$ a vector function.

EXAMPLE 2: Sketch the following curves described by the vector function. Include the direction of the curve as t increases.

(a) $\vec{r}(t) = \langle t - 1, 2 - 3t \rangle$

(b) $\vec{r}(t) = \langle 2 + \cos t, 1 + \sin t \rangle$

<u>Lines</u>: A **vector equation of the line** passing through the point $r_0 = (x_0, y_0)$ and parallel to the vector $\vec{v} = \langle v_1, v_2 \rangle$ is given by $\vec{r}(t) = \vec{r_0} + t\vec{v}$. From this vector equation, we can obtain the parametric equations of the line as follows:

$\vec{r}(t) = \vec{r_0} + t\vec{v} = \langle x_0, y_0 \rangle + t \langle v_1, v_2 \rangle$

$= \langle x_0, y_0 \rangle + \langle tv_1, tv_2 \rangle$

$= \langle x_0 + tv_1, y_0 + tv_2 \rangle \Rightarrow x = x_0 + tv_1$ and $y = y_0 + tv_2$ are **parametric equations of the line**.

<u>Illustration</u>:

EXAMPLE 3: Find a vector equation of the line parallel to the vector $\langle 1, 4 \rangle$ and passing through the point $(-1, 5)$.

EXAMPLE 4: Find parametric equations for the line with slope $\frac{4}{3}$ and passing through the point $(2, -5)$.

EXAMPLE 5: Find a vector equation of the line passing through the points $(1, 2)$ and $(-1, 4)$.

EXAMPLE 6: Consider the line $2x + 3y = 5$.

(a) Find a vector parallel to the line.

(b) Find a vector perpendicular to the line.

EXAMPLE 7: An object is moving in the xy-plane and its position after t seconds is given by $\vec{r}(t) = \langle t+4, t^2+2 \rangle$.

(a) Find the position of the object at time $t = 2$.

(b) At what time does the object reach the point $(7, 11)$?

(c) Eliminate the parameter to obtain a cartesian equation.

EXAMPLE 8: Consider the lines $\vec{r}(t) = \langle -4+2t, 5+t \rangle$ and $\vec{s}(w) = \langle 2+3w, 4-6w \rangle$. Determine whether the lines are parallel, perpendicular or neither. If they are not parallel, find the intersection point.