Section 2.2: Limits

Motivation of limits Consider $f(x) = \dfrac{\sin(x)}{x}$. This function is not defined at $x = 0$. However, we can get as close to 0 as we wish, provided we never *reach* 0. As x gets closer and closer to 0, we want to know if $f(x)$ approaches what we call a limiting value. To investigate this, we will evaluate $f(x)$ for x close to 0.

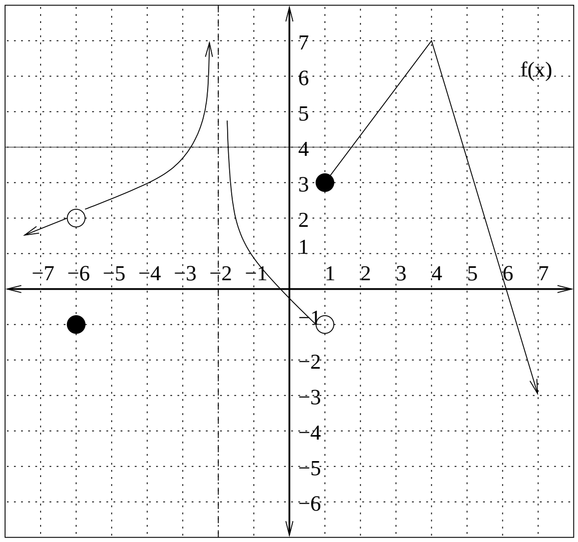
Definition:

(i) If $\lim\limits_{x\to a^-} f(x) = L$ and $\lim\limits_{x\to a^+} f(x) = L$, then $\lim\limits_{x\to a} f(x)$ exixts and $\lim\limits_{x\to a} f(x) = L$.

(ii) If $\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$, then $\lim\limits_{x\to a} f(x)$ does not exist.

Illustration

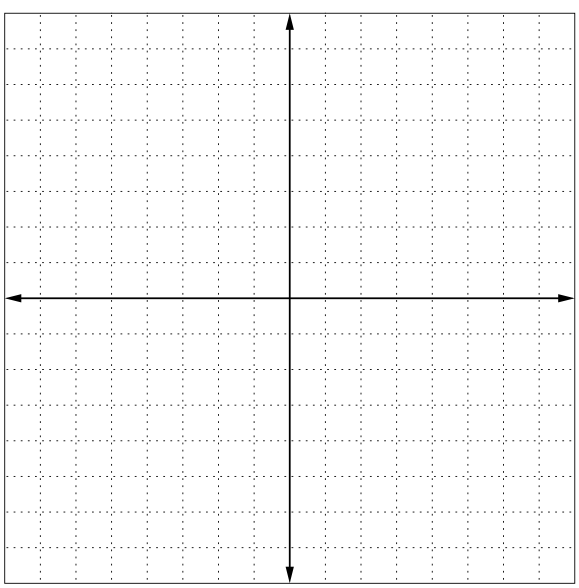
Graphical Limits



Limits of piece-wise defined functions

EXAMPLE 1 : $f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ 4 & \text{if } x = 1 \\ 4-x & \text{if } x > 1 \end{cases}$

Sketch the graph of $f(x)$ and find all values of x for which the limit does not exist.



Infinite Limits and vertical asymptotes: If $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$, then we say $x = a$ is a vertical asymptote of $f(x)$.

Discussion: What characteristics must $f(x)$ have in order to contain vertical asymptotes in its graph?

EXAMPLE 2: Find the infinite limit:

(a) $\lim_{x\to 5^-} \frac{6}{x-5}$

(b) $\lim_{x\to 5^+} \frac{6}{x-5}$

(c) $\lim_{x\to 5} \frac{6}{x-5}$

(d) $\lim_{x \to 0^+} \frac{x-1}{x^2(x+2)}$

(e) $\lim_{x \to 0^-} \frac{x-1}{x^2(x+2)}$

(f) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$

(g) $\lim_{x \to \pi^-} \csc x$

EXAMPLE 3: Find all vertical asymptotes for $f(x) = \dfrac{x+1}{x^2 - 2x - 3}$. For each vertical asymptote, describe the behavior of $f(x)$ near the asymptote.