

Section 2.3: Limit Laws

Limit Laws If $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exist and c is any constant. Then:

$$(1) \lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$(2) \lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$$

$$(3) \lim_{x \rightarrow a} f(x)g(x) = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$$

$$(4) \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \text{ provided } \lim_{x \rightarrow a} g(x) \neq 0. \text{ If } \lim_{x \rightarrow a} g(x) = 0, \text{ try to simplify}$$

$$\frac{f(x)}{g(x)}$$

$$(5) \lim_{x \rightarrow a} (f(x))^n = \left(\lim_{x \rightarrow a} f(x) \right)^n$$

$$(6) \lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}. \text{ In the event } n \text{ is even, we must have } \lim_{x \rightarrow a} f(x) > 0.$$

EXAMPLE 1: Find the limit. If the limit does not exist, support your answer by evaluating left and right handed limits.

$$(a) \lim_{x \rightarrow -2} (x^2 + x + 1)^5$$

$$(b) \lim_{x \rightarrow 4^-} \sqrt{16 - x^2}$$

$$(c) \lim_{x \rightarrow 1} \frac{x^4 + x^2 - 6}{x^4 + 2x + 3}$$

$$(d) \lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$$

$$(e) \lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3}$$

$$(f) \lim_{h \rightarrow 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$$

$$(g) \lim_{x \rightarrow 2} \frac{x - 4}{(x - 2)^2}$$

$$(h) \lim_{x \rightarrow 1} \left(\frac{1}{x - 1} - \frac{2}{x^2 - 1} \right)$$

$$(i) \lim_{x \rightarrow 4} \frac{x - 4}{|x - 4|}$$

$$(j) \lim_{x \rightarrow -3} \left(\frac{|x + 3|}{x^2 + 3x} \right)$$

$$(k) \lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$

$$(l) \text{ If } f(x) = \begin{cases} 4 + 5x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ 3 & \text{if } x = 1 \\ 4 - x & \text{if } x > 1 \end{cases}, \text{ find } \lim_{x \rightarrow -1} f(x) \text{ and } \lim_{x \rightarrow 1} f(x).$$

Squeeze Theorem If $f(x) \leq g(x) \leq h(x)$ for all x in an interval containing $x = a$ (except possibly at $x = a$) and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$.

EXAMPLE 2: Given $3x \leq f(x) \leq x^3 + 2$ for all x in the interval $(0, 3)$, find $\lim_{x \rightarrow 1} f(x)$.

EXAMPLE 3: Find $\lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{x}\right)$