Section 2.6: Limits at Infinity; Horizontal Asymptotes

Definition If $\lim_{x\to\infty} f(x) = L$, or if $\lim_{x\to-\infty} f(x) = L$, then we say $f(x)$ has a **horizontal asymptote** at $y = L$.

EXAMPLE 1: Find the limits:

(i) $\lim_{x\to\infty} \dfrac{7x^3 + 4x}{2x^3 - x^2 + 3}$

(ii) $\lim_{t\to\infty} \dfrac{t^4 - t^2 + 1}{t^5 + t^3 - t}$

(iii) $\lim_{x \to -\infty} \dfrac{x^4 + 2x + 3}{x(x^2 - 1)}$

(iv) $\lim_{x \to \infty} \dfrac{\sqrt{1 + 4x^2}}{4 + x}$

(v) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 + 4x}}{4x + 1}$

(vi) $\lim_{x\to\infty}(\sqrt{x^2+3x+1}-x)$

(vii) $\lim_{x\to-\infty}(x+\sqrt{x^2+2x})$

EXAMPLE 2: Find the equation of all vertical and horizontal asymptotes for:

(i) $f(x) = \dfrac{x+3}{x^2+7x+12}$

(ii) $f(x) = \dfrac{x}{\sqrt{x^2+1}}$