Section 2.7: Tangents, Velocities, and Other Rates of Change

Tangents We want to develope a formula for the slope of the tangent line to the graph of $f(x)$ at $x = a$.

EXAMPLE 1: Find the slope of the tangent line to the graph of $f(x) = x^2 + 2x$ at the point $(1, 3)$.

EXAMPLE 2: Find the equation of the tangent line to the graph of $f(x) = \dfrac{1}{x+2}$ at $x = 3$.

EXAMPLE 3: (a) Find a vector tangent to the curve $\vec{\mathbf{r}}(t) = \langle t^2 + t, 5t \rangle$ at $t = 2$.

(b) Find parametric equations of the tangent line to $\vec{\mathbf{r}}(t)$ at $t = 2$

(c) Find a cartesian equation of this tangent line.

Velocities If $f(t)$ is the position of an object at time t, then

(a) The **Average Velocity** of the object from $t = a$ to $t = b$ is

$$\frac{f(b) - f(a)}{b - a}$$

(b) The **Instantaneous Velocity** of the object at time $t = a$ is

$$v(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

EXAMPLE 4: The position (in meters) of an object moving in a straight path is given by $s(t) = t^2 - 8t + 18$, where t is measured in seconds.

(i) Find the average velocity over the time interval $[3, 4]$.

(ii) Find the instantaneous velocity at time $t = 3$.

Other Rates of Change Let $f(x)$ be a function.

(a) The **Average Rate of change** of $f(x)$ from $x = a$ to $x = b$ is

$$\frac{f(b) - f(a)}{b - a}$$

(b) The **Instantaneous Rate of Change** of $f(x)$ at $x = a$ is

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

EXAMPLE 5: If $f(x) = \sqrt{x}$, find

(i) the average rate of change of $f(x)$ from $x = 4$ to $x = 9$.

(ii) The instantaneous rate of change of $f(x)$ at $x = 4$.

EXAMPLE 6: The table below gives the population of a certain city from years 1990 to 1996.

Year	Population (in thousands)
1990	105
1991	110
1992	117
1993	126
1994	137
1995	150
1996	164

(i) Find the average growth rate from 1992 to 1994

(ii) Estimate the instantaneous growth rate in year 1992.