Section 3.1: Derivatives

Definition: The **Derivative** of $f(x)$ at $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

EXAMPLE 1: Find the derivative of

(i) $f(x) = \dfrac{x}{x+1}$ at $x = 2$.

(ii) $f(x) = \sqrt{x}$ at $x = 3$.

How to interpret the derivative: $f'(a)$ measures:

(a) The slope of the tangent line to the graph of $f(x)$ at $x = a$

(b) The instantaneous rate of change of $f(x)$ at $x = a$

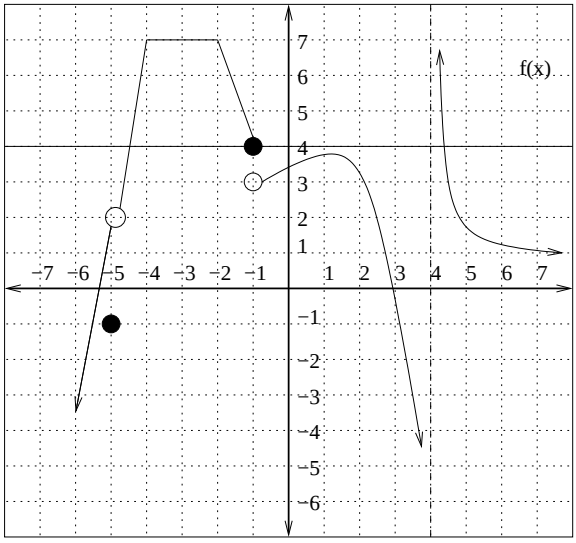
(c) The instantaneous velocity at $x = a$.

EXAMPLE 2: If $f(x) = x^2 - x$, find $f'(-1)$ and use it to find the slope of the tangent line to the graph of $f(x)$ at $x = -1$.

EXAMPLE 3: Recall the surface area of a sphere is given by $A = 4\pi r^2$. Find the average rate of change of the area from $r = 1$ to $r = 2$. Find the instantaneous rate of change of the area at $r = 1$.

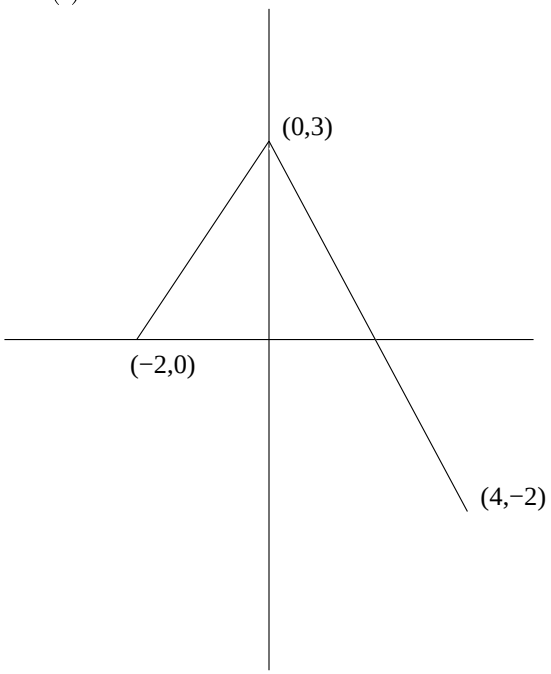
Definition: Let $f(x)$ be a function. We say $f(x)$ is **differentiable** at $x = a$ if $f'(a)$ exists.

EXAMPLE 4: Refer to the graph below to determine where $f(x)$ is not differentiable.

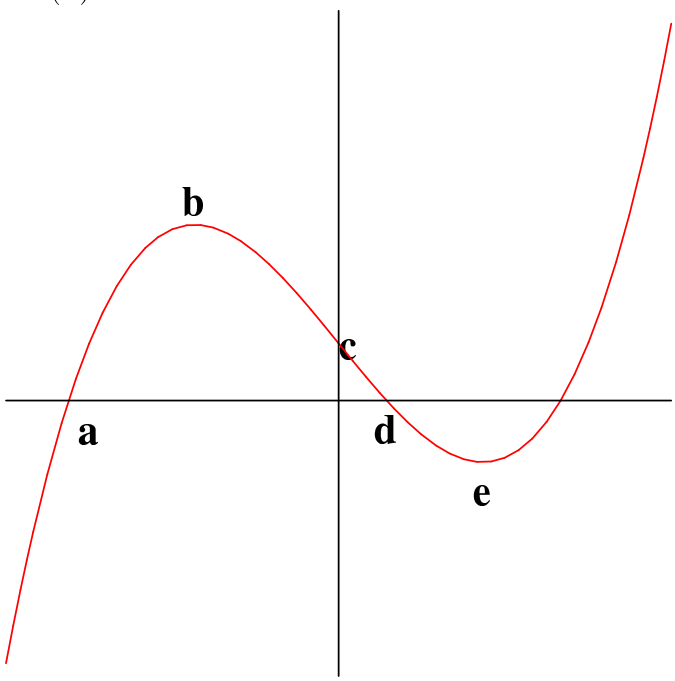


EXAMPLE 5: Given the graphs of $f(x)$ below, sketch the graph of the derivative.

(i)



(ii)



Definition The **Derivative** of $f(x)$ is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

EXAMPLE 6: Find $f'(x)$ if $f(x) = \sqrt{x-6}$ using the defintion of the derivative.

EXAMPLE 7: The limits below represent the derivative of some function $f(x)$ at some number a. Identify $f(x)$ and a for each limit.

(i) $\lim_{h\to 0} \dfrac{(1+h)^{2011}-1}{h}$

(ii) $\lim_{x\to 3\pi} \dfrac{\cos x + 1}{x - 3\pi}$