

Section 3.10: Related Rates

Related Rates: In this section, we have two or more quantities that are changing with respect to time. Our first goal is to find a formula that relates the quantities. Then we will solve for the desired rate of change.

EXAMPLE 1: Suppose both x and y are changing size with respect to time t (assume x and y are both positive). At the instant when $x = 4$, the rate of change of x with respect to t is -2 . If x and y are related by the equation $x^3 + 4y^2 = 84$, find the corresponding change in y at this same instant.

EXAMPLE 2: A stone is thrown into a still pond and circular ripples move out. The radius of the disturbed region increases at a rate of $3ft/sec$. Find the rate at which the area of the disturbed region is increasing when the farthest ripple is $20ft$ from the place at which the rock struck the pond.

EXAMPLE 3: A storm is 50 miles offshore and its path is perpendicular to a straight shoreline. It is approaching the shore at a rate of $4mph$. A van traveling along the shoreline wants to stay exactly 50 miles from the storm and remain along the shoreline. The van starts at the point on the shoreline in the path of the storm.

(i) Determine a formula for the speed that the van must maintain to remain 50 miles from the storm.

(ii) Find the speed of the van when the storm is 40 miles from the shore.

EXAMPLE 4: A water tank has the shape of an inverted circular cone with radius 5 meters and height 7 meters. A solution is being poured into the cone in such a way that the height of the fluid is increasing at a rate of $\frac{1}{2}$ meter per minute. At what rate is the volume increasing when the height is 4 meters?

EXAMPLE 5: A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2ft/s , how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians?

EXAMPLE 6: The length of a rectangle is increasing at the rate of 2 feet per second, while the width is increasing at the rate of 1 foot per second. How fast is the area increasing when the length is 5 feet and the area is 50 square feet?

EXAMPLE 7: If a snowball melts so that its surface area decreases at a rate of $1 \frac{\text{cm}^2}{\text{min}}$, find the rate at which the diameter decreases when the diameter is 10 cm.

EXAMPLE 8: At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr. Ship B is sailing north at 25 km/hr. How fast is the distance between them changing at 4:00 pm?

EXAMPLE 9: A trough is 10 feet long and its ends have the shape of isosceles triangles that are 3 feet across the top and have a height of 1 foot. If the trough is filled with water at a rate of 12 cubic feet per minute, how fast is the water level rising when the water is 6 inches deep?

EXAMPLE 10: A plane flying with a constant speed of 300 km per hour passes over a radar station at an altitude of 1 km and climbs at an angle of 30° . At what rate is the distance from the plane to the radar station increasing 1 minute later?