

Section 3.2: Differentiation Formulas

Differentiation Formulas:

- (1) Constant rule: If $f(x) = c$, where c is a constant, then $f'(x) = 0$.
- (2) Power rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$
- (3) Constant times a function rule: $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$
- (4) Sum/Difference rule: If $f(x) = g(x) \pm h(x)$, then $f'(x) = g'(x) \pm h'(x)$
- (5) Product rule: If $f(x) = g(x)h(x)$, then $f'(x) = g(x)h'(x) + g'(x)h(x)$
- (6) Quotient rule: If $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$

EXAMPLE 1: Find the derivative of the following functions.

(a) $g(x) = x^5 + 8x^2 - 16x + 2 - \pi^2$

(b) $f(t) = (1 - \sqrt{t})^2$

(c) $H(s) = \left(\frac{s}{2}\right)^5$

$$(d) F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$$

$$(e) y = (x^3 - x^2 - 2x + 1)(5x^4 - 20x^3 + 5x + 3)$$

$$(f) f(u) = \frac{1 - u^2}{1 + u^2}$$

EXAMPLE 2: If $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$, find the value of $(fg)'(5)$.

EXAMPLE 3: Find the equation of the tangent line to the graph of $f(x) = x + \sqrt{x}$ at the point $(1, 2)$.

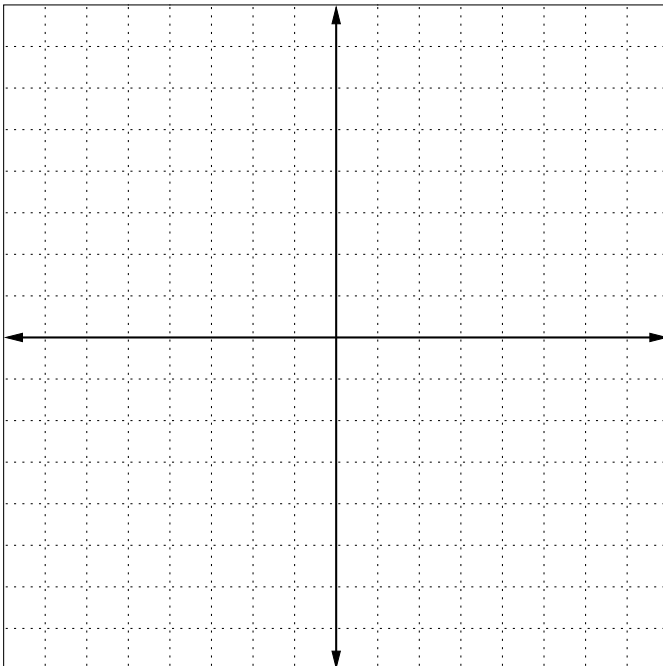
EXAMPLE 4: At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

EXAMPLE 5: Show there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the equation of these tangent lines.

EXAMPLE 6: If $f(x) = \begin{cases} -1 - 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ x & \text{if } x \geq 1 \end{cases}$

(i) Find where $f(x)$ is not differentiable.

(ii) Sketch the graph of $f(x)$ and $f'(x)$ on the same axis.



EXAMPLE 7: If $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$, find the value of m and b that make $f(x)$ differentiable everywhere.

EXAMPLE 8: If $\vec{r}(t) = \langle t^2 + 2t, t^3 + 3t^2 \rangle$ is the position of a moving object at time t , where the position is measured in feet and the time in seconds, find the velocity and speed at time $t = 1$