

Section 3.3: Rates of Change in the Natural and Social Sciences

EXAMPLE 1: A particle moves according to the equation of motion $s(t) = 4t^3 - 9t^2 + 6t + 2$, where $s(t)$ is measured in meters and t in seconds.

(a) Find the velocity at time t .

$$v(t) = 12t^2 - 18t + 6$$

(b) When is the particle at rest?

at rest when $v(t) = 0$

$$12t^2 - 18t + 6 = 0$$

$$6(2t^2 - 3t + 1) = 0$$

$$6(2t-1)(t-1) = 0$$

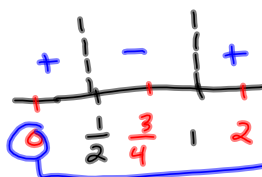
$$t = \frac{1}{2}, t = 1 \text{ seconds}$$

(c) When is the particle moving in the positive direction?

solve $v(t) > 0$

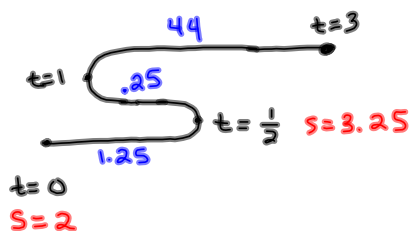
$$\left[0, \frac{1}{2}\right) \cup (1, \infty)$$

$$0 \leq t < \frac{1}{2}, t > 1$$



$$v(t) = 6(2t-1)(t-1)$$

(d) Draw a diagram that represents the motion of the particle.



$$s(t) = 4t^3 - 9t^2 + 6t + 2$$

(e) Find the distance traveled in the first 3 seconds.

$$1.25 + .25 + 4.4$$

$$45.5 \text{ m}$$

$$s(0) = 2$$

$$s\left(\frac{1}{2}\right) = 3.25$$

$$s(1) = 3$$

$$s(3) = 47$$

EXAMPLE 2: A ball is thrown vertically upward with a velocity of 80 feet per second. The height after t seconds is given by $h(t) = 80t - 16t^2$. What is the maximum height of the ball?

$$\text{max height: } v(t) = 0$$

$$80 - 32t = 0$$

$$32t = 80$$

$$t = 2.5 \text{ seconds}$$

$$h(2.5) = 80(2.5) - 16(2.5)^2$$

$$= \boxed{100 \text{ feet}}$$