

Section 3.4: Derivatives of Trigonometric Functions

Two special limits:

$$(1) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

EXAMPLE 1: Find the limit:

$$(i) \lim_{x \rightarrow 0} \frac{\sin x}{3x}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\sin 9x}{7x}$$

$$(iii) \lim_{x \rightarrow 0} \frac{\sin 8x}{\sin 7x}$$

$$(iv) \lim_{x \rightarrow 0} \frac{\tan^2 4x}{x^2}$$

$$(v) \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x}$$

Derivatives of Trigonometric Functions:

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

EXAMPLE 2: Find the derivative:

(i) $f(x) = \sec^2 x + 4 \tan x + x\sqrt{x}$

(ii) $g(t) = \frac{2 \cos t + 1}{\cot t + t}$

EXAMPLE 3: Find the equation of the tangent line to the graph of $f(x) = 2 \sin x$ at $x = \frac{\pi}{3}$.

EXAMPLE 4: For what value(s) of x does the graph of $f(x) = x + 2 \sin x$ have a horizontal tangent?