Section 3.5: Chain Rule

Chain Rule: We use the chain rule when we are differentiating a function written as a composition of functions, that is $f(x) = g(h(x))$. Then $f'(x) = g'(h(x))h'(x)$.

EXAMPLE 1: Find the derivative:

(i) $f(x) = \sin(2x) + \cot(5x^2)$

(ii) $g(t) = \tan(\cos(t))$

(iii) $h(w) = \sec(\cos(\sin(4w^2)))$

Generalized Power Rule: If $f(x) = (g(x))^n$, then $f'(x) = n\,(g(x))^{n-1}\,g'(x)$

EXAMPLE 2: Find the derivative:

(i) $f(x) = \dfrac{1}{(x^2+5x+4)^{10}}$

(ii) $g(x) = x^3(\sqrt{x}+5)^3$

(iii) $f(x) = \sin(3x) + \sin^3(x)$

(iv) $h(t) = \sqrt{\cos(\sin^2 t)}$

(v) $g(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

EXAMPLE 3: Find the equation of the tangent line to the graph of $f(x) = 8\sqrt{4 + 3x}$ at $x = 4$.

EXAMPLE 4: Suppose $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$ and $v'(2) = 6$. Find $w'(0)$.

EXAMPLE 5: If $F(x) = f(\cos x)$, $G(x) = \cos(f(x))$ and $H(x) = [f(\sin x)]^3$, find $F'(x)$ and $G'(x)$ and $H'(x)$.

EXAMPLE 6: Find all points on the curve $y = \sin(2x) + \cos(2x)$ where the tangent line is horizontal.