

Section 3.6: Implicit Differentiation

Why we differentiate Implicitly: Suppose $y = f(x)$. In this case, we say y is an *explicit* function of x and we can therefore differentiate as usual: $\frac{dy}{dx} = f'(x)$. In this section, we investigate how to differentiate if y cannot be written as an explicit function of x , that is y is *implicitly* defined as a function of x .

EXAMPLE 1: Find $\frac{dy}{dx}$ for the following equations:

(i) $y^3 = 2x^2 + y^4$

(ii) $2y^2 + xy = x^2 + 3$

$$(iii) \ x \sin y + \cos(2y) = \cos y$$

$$(iv) \ (x^2 + y^2)^3 = 2y^4 + 6x^2$$

EXAMPLE 2: Find the tangent line to the ellipse $\frac{x^2}{9} + \frac{y^2}{36} = 1$ at the point $(-1, 4\sqrt{2})$.

Definition: We say two curves are orthogonal if their tangent lines are perpendicular at every point of intersection.

EXAMPLE 3: Prove the curves $x^2 - y^2 = 5$ and $4x^2 + 9y^2 = 72$ are orthogonal.

EXAMPLE 4: If $[g(x)]^2 + 12x = x^2g(x)$ and $g(3) = 4$, find $g'(3)$.

EXAMPLE 5: Regard y as the independent variable and x as the dependent variable, and use implicit differentiation to find $\frac{dx}{dy}$ for the equation $(x^2 + y^2)^2 = 2x^2y$.