# Section 3.6: Implicit Differentiation

**Why we differentiate Implicitly:** Suppose $y = f(x)$. In this case, we say $y$ is an *explicit* function of $x$ and we can therefore differentiate as usual: $\frac{dy}{dx} = f'(x)$. In this section, we investigate how to differentiate if $y$ cannot be written as an explicit function of $x$, that is $y$ is *implicitly* defined as a function of $x$.

*EXAMPLE 1:* Find $\frac{dy}{dx}$ for the following equations:

(i) $y^3 = 2x^2 + y^4$

(ii) $2y^2 + xy = x^2 + 3$

(iii) $x \sin y + \cos(2y) = \cos y$

(iv) $(x^2 + y^2)^3 = 2y^4 + 6x^2$

*EXAMPLE 2:* Find the tangent line to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ at the point $(-1, 4\sqrt{2})$.

**Definition:** We say two curves are orthogonal if their tangent lines are perpendicular at every point of intersection.

*EXAMPLE 3:* Prove the curves $x^2 - y^2 = 5$ and $4x^2 + 9y^2 = 72$ are orthogonal.

*EXAMPLE 4:* If $[g(x)]^2 + 12x = x^2 g(x)$ and $g(3) = 4$, find $g'(3)$.

*EXAMPLE 5:* Regard $y$ as the independent variable and $x$ as the dependent variable, and use implicit differentiation to find $\dfrac{dx}{dy}$ for the equation $(x^2 + y^2)^2 = 2x^2 y$.