Section 3.8: Higher Derivatives

**Defintion:** If $y = f(x)$, then the second derivative of $f(x)$ is the derivative of the first derivative. We denote the second derivative as $y'' = (f'(x))' = f''(x)$. Similarly, the third derivative is the derivative of the second derivative, denoted by $f'''(x)$. In general, the $n^{th}$ derivative of $f(x)$ is denoted by $f^{(n)}(x)$.

*EXAMPLE 1:* Find the second derivative of $f(\theta) = \theta \sin(\theta)$.

*EXAMPLE 2:* Find the $f^{(81)}(x)$ for $f(x) = \cos(10x)$.

*EXAMPLE 3:* Find a general formula for the $n^{th}$ derivative for $f(x) = \dfrac{1}{x}$.

*EXAMPLE 4:* If $s(t) = 2t^3 - 7t^2 + 4t + 1$ is the position of a moving object at time $t$, where $s(t)$ is measured in feet and $t$ is measured in seconds, find:

(i) The velocity at time $t$.

(ii) The acceleration at the times when the velocity is zero.

*EXAMPLE 5:* If $\mathbf{r}(t) = \left\langle \frac{t}{2}, t^2 \right\rangle$:

(i) Sketch the curve.

(ii) Plot the position, tangent and acceleration vectors at at the point corresponding to $t = 2$.

*EXAMPLE 6:* Find $f''(x)$ if $f(x) = g(x^3) + (g(x))^3$.

*EXAMPLE 7:* Find $y''$ by implicit differentiation for the equation $x^2 + 6xy^2 = 8$.