Section 3.8: Higher Derivatives

Defintion: If $y = f(x)$, then the second derivative of $f(x)$ is the derivative of the first derivative. We denote the second derivative as $y'' = (f'(x))' = f''(x)$. Similarly, the third derivative is the derivative of the second derivative, denoted by $f'''(x)$. In general, the n^{th} derivative of $f(x)$ is denoted by $f^{(n)}(x)$.

EXAMPLE 1: Find the second derivative of $f(\theta) = \theta \sin(\theta)$.

EXAMPLE 2: Find the $f^{(81)}(x)$ for $f(x) = \cos(10x)$.

EXAMPLE 3: Find a general formula for the n^{th} derivative for $f(x) = \dfrac{1}{x}$.

EXAMPLE 4: If $s(t) = 2t^3 - 7t^2 + 4t + 1$ is the position of a moving object at time t, where $s(t)$ is measured in feet and t is measured in seconds, find:

(i) The velocity at time t.

(ii) The acceleration at the times when the velocity is zero.

EXAMPLE 5: If $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$:

(i) Sketch the curve.

(ii) Plot the position, tangent and acceleration vectors at time $t = \dfrac{\pi}{4}$.

EXAMPLE 6: Find $f''(x)$ if $f(x) = g(x^3) + (g(x))^3$.

EXAMPLE 7: Find y'' by implicit differentiation for the equation $x^2 + 6xy^2 = 8$.