Section 3.9: Slopes and Tangents to Parametric Curves

Derivatives of Parametric Curves: If $x = f(t)$ and $y = g(t)$, then $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$. This gives us a way to find the slope of the tangent line to the parametric curve at $t = t_0$: $m = \dfrac{dy}{dx}\Big|_{t=t_0}$.

EXAMPLE 1: Find $\dfrac{dy}{dx}$ if $x = (3t-1)^2$ and $y = t\sqrt{t}$.

EXAMPLE 2: If $x = 1 - t^3$ and $y = t^2 - 3t + 1$, find an equation of the tangent line corresponding to $t = 2$.

EXAMPLE 3: If $x = 2t + 3$ and $y = t^2 + 2t$, find the equation of the tangent line at the point $(5, 3)$.

EXAMPLE 4: If $x = t^3 - 3t^2$ and $y = t^3 - 3t$, find all points on the curve where the tangent line is vertical or horizontal.

EXAMPLE 5: Show the curve $x = \cos t$ and $y = (\sin t)(\cos t)$ has two tangents at $(0, 0)$. Find the equations of these tangent lines.

EXAMPLE 6: At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent line parallel to the line with equations $x = -7t$, $y = 12t - 5$?