# Section 4.2: Inverse Functions

**Definition**: We say $f(x)$ is one-to-one provided whenever $f(x_1) = f(x_2)$, $x_1 = x_2$.

*EXAMPLE 1*: Prove $f(x) = x^2 - 2x + 5$ is not one-to-one.

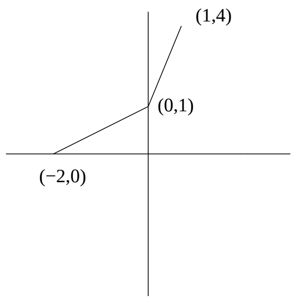
*EXAMPLE 2*: Prove $f(x) = 5 - 4x^3$ is one-to-one.

*EXAMPLE 3*: Prove $f(x) = \dfrac{x-2}{x+2}$ is one-to-one.

**Definition**: Let $f(x)$ be a one-to-one function with domain $D$ and range $R$. Then the inverse exists, denoted by $f^{-1}(x)$. Furthermore, the domain of $f^{-1}$ = range of $f = R$ and the range of $f^{-1}$ = domain of $f = D$. Moreover,

$$f(x) = y \iff f^{-1}(y) = x$$

*EXAMPLE 4*: Given the graph of $f$ below, sketch the graph of $f^{-1}$.




*EXAMPLE 5*: Find the inverse.

(a) $f(x) = 5 - 4x^3$

(b) $f(x) = \sqrt{4 - 2x}$

(c) $f(x) = \dfrac{2x+1}{1-3x}$

**Theorem**: Suppose $f$ is a one-to-one differentiable function with inverse function $g = f^{-1}$. Then $g$ is differentiable and $g'(a) = \dfrac{1}{f'(g(a))}$.

*EXAMPLE 6*: Suppose $g$ is the inverse of $f$ and $f(2) = 3$, $f'(2) = 7$, $f(3) = 4$ and $f'(3) = \dfrac{1}{2}$. Find $g'(3)$.

*EXAMPLE 7*: Suppose $g$ is the inverse of $f$. Find $g'(4)$ if $f(x) = 3 + x + e^x$.

*EXAMPLE 8*: Suppose $g$ is the inverse of $f$. Find $g'(2)$ if $f(x) = \sqrt{x^3 + x^2 + x + 1}$.