Section 4.2: Inverse Functions

Definition: We say $f(x)$ is one-to-one provided whenever $f(x_1) = f(x_2)$, $x_1 = x_2$.

EXAMPLE 1: Prove $f(x) = x^2 - 2x + 5$ is not one-to-one.

EXAMPLE 2: Prove $f(x) = 5 - 4x^3$ is one-to-one.

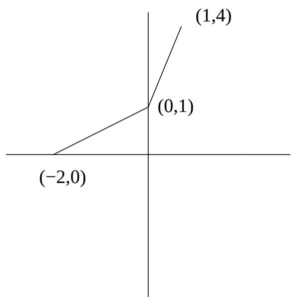
EXAMPLE 3: Prove $f(x) = \dfrac{x-2}{x+2}$ is one-to-one.

EXAMPLE 4: How can we restrict the domain of $f(x) = \cos x$ to make it one-to-one?

<u>Definition</u>: Let $f(x)$ be a one-to-one function with domain D and range R. Then the inverse exists, denoted by $f^{-1}(x)$. Furthermore, the domain of f^{-1} = range of $f = R$ and the range of f^{-1} = domain of $f = D$. Moreover,

$$f(x) = y \iff f^{-1}(y) = x$$

EXAMPLE 5: Given the graph of f below, sketch the graph of f^{-1}.



EXAMPLE 6: Find the inverse and find the domain and range of the inverse.

(a) $f(x) = 5 - 4x^3$

(b) $f(x) = \dfrac{2x+1}{1-3x}$

(c) $f(x) = x^2 + x$, for $x \geq -\dfrac{1}{2}$

Theorem: Suppose f is a one-to-one differentiable function with inverse function $g = f^{-1}$. Then g is differentiable and $g'(a) = \dfrac{1}{f'(g(a))}$.

EXAMPLE 7: Suppose g is the inverse of f and $f(2) = 3$, $f'(2) = 7$, $f(3) = 4$ and $f'(3) = \dfrac{1}{2}$. Find $g'(3)$.

EXAMPLE 8: Suppose g is the inverse of f. Find $g'(4)$ if $f(x) = 3 + x + e^x$.

EXAMPLE 9: Suppose g is the inverse of f. Find $g'(2)$ if $f(x) = \sqrt{x^3 + x^2 + x + 1}$.