Section 4.3: Logarithmic Functions

Definition: If $a > 0$ and $a \neq 1$, $f(x) = a^x$ is a one to one function, hence has an inverse, called the logarithm with base a. More specifically, If $a^x = y$, then $\log_a y = x$. Note in particular, since $\log_a x$ and a^x are inverses of each other, it follows that $\log_a a^x = x$ and $a^{\log_a x} = x$.

EXAMPLE 1: Evaluate the following:

(a) $\log_2 8$

(b) $\log_3 \frac{1}{81}$

(c) $\log_{16} 4$

EXAMPLE 2: Sketch the graph of $f(x) = \log_4 x$. What is the domain and range?

Definition: We define the natural logarithm to be $\log_e x$ denoted by $\ln x$. We define the common logarithm to be $\log_{10} x$, dentoed by $\log x$.

EXAMPLE 3: Evaluate the following:

(a) $\log \frac{1}{100}$

(b) $\ln \sqrt{e}$

EXAMPLE 4: Solve for x:

(a) $2\log(x+1) = 3$

(b) $e^{3-x} + 8 = 14$

(c) $4^{\log_4 3} + \ln(4x-2) = 15$

EXAMPLE 5: Find the domain of $f(x) = \log(x^2 - 3x + 2)$.

EXAMPLE 6: Find the limit:

(a) $\lim_{x \to \infty} \ln(x^2 - x)$

(b) $\lim_{x \to 9^+} \log_2(x - 9)$

(c) $\lim_{x \to \frac{\pi}{2}^-} \log(\tan x)$

(d) $\lim_{x \to 0^+} \log(\cos x)$

<u>Properties of Logarithms</u>:

- $\log_a(MN) = \log_a M + \log_a N$
- $\log_a \dfrac{M}{N} = \log_a M - \log_a N$
- $\log_a M^r = r \log_a M$
- $\log_a a^x = x$
- $a^{\log_a x} = x$
- If $\log_a M = \log_a N$, then $M = N$.

EXAMPLE 7: Write the following as a single logarithm:
$\frac{1}{2}\ln x + b\ln y - c\ln z - \ln(x^2+1)$.

EXAMPLE 8: Solve the following equations for x.

(a) $\ln x - \ln(x-1) = 1$

(b) $\log x + \log(x+1) = \log 6$

EXAMPLE 9: Find the limit:

a.) $\lim_{x\to\infty} \left(\ln(3x^2 + x) - \ln(2x^2 - x)\right)$

b.) $\lim_{x\to\infty} \left(\ln(3x^2 + x) - \ln(2x^4 - x)\right)$

EXAMPLE 10: Find the inverse of the function:

(a) $f(x) = \ln(x+2)$

(b) $f(x) = \dfrac{10^x}{10^x + 1}$

EXAMPLE 11: Find the point where the tangent line to the curve $y = e^{-x}$ is perpendicular to the line $2x - y = 8$.

<u>Change of Base formula</u> $\log_a x = \dfrac{\log_b x}{\log_b a}$. Specifically, to change to base e:

$$\log_a x = \frac{\ln x}{\ln a}$$

EXAMPLE 12: Evaluate $\log_2 5$ to 4 decimal places.