Section 4.4: Derivatives of Logarithmic Functions

EXAMPLE 1: Using Implicit Differentiation, prove $\frac{d}{dx}\ln x = \frac{1}{x}$.

Note: The chain rule states $\frac{d}{dx}\ln(g(x) = \frac{g'(x)}{g(x)}$

EXAMPLE 2: Find the derivative

(a) $f(x) = \cos(\ln x)$

(b) $y = \ln(1 + \ln x)$

(c) $f(u) = \ln\sqrt{\frac{3u+2}{3u-2}}$

EXAMPLE 3: Using Implicit Differentiation, prove $\frac{d}{dx}a^x = (\ln a)a^x$.

Note: The chain rule states $\frac{d}{dx}a^{g(x)} = g'(x)\ln(a)a^{g(x)}$

EXAMPLE 4: Find the derivative of $f(x) = \tan^5 x + 5^{\tan x}$

EXAMPLE 5: Using the change of base formula, prove $\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$

Note: The chain rule states $\frac{d}{dx}\log_a g(x) = \frac{g'(x)}{g(x)\ln a}$

EXAMPLE 6: Find the derivative of $f(x) = \log_3(5 - x^4)$.

Logarithmic Differentiation: We use this technique in two situations:

(i) When we are differentiating a function of the form $y = f(x)^{g(x)}$

(ii) When we wish to split a product or quotient, thereby making the derivative easier to compute.

EXAMPLE 7: Find the derivative of

(a) $f(x) = (\sin x)^{\cos x}$

(b) $y = \dfrac{e^x(x^2+2)^3}{(x+1)^4(x^2+3)^2}$