Section 4.5: Exponential Growth and Decay

Definition: If $y(t)$ is the value of a quantity at time t and if the rate of change of y with respect to t is proportional to its size $y(t)$ at any time, [That is $\frac{dy}{dt} = ky$], then the quantity $y(t)$ at time t is given by

$$y(t) = y_0 e^{kt}$$

where y_0 is the initial quantity and k is a constant. Given information, your primary goal is to find k.

EXAMPLE 1: A bacteria culture starts with 4000 bacteria and the population triples every half-hour.

(i) Find an expression for the number of bacteria after t hours.

(ii) Find the number of bacteria after 20 minutes.

(iii) Find the rate of growth after 20 minutes.

EXAMPLE 2: Def: The **half-life** of a substance is the amount of time it takes for half of the substance to disintegrate. Polonium-210 has a half-life of 140 days.

(i) If a sample has a mass of 200 mg, find a formula for the mass that remains after t days.

(ii) When will the mass be reduced to 10 mg?

EXAMPLE 3: After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222? How long will it take the sample to decay to 10% of its original amount?

EXAMPLE 4: A curve passes through the point $(0, 7)$ and has the property that the slope of the curve at every point p is half the y-coordinate of p. Find the equation of the curve.

EXAMPLE 5: The rate of change of atmospheric pressure P with respect to altitude h is proportional to P, provided that the temperature is constant. At a specific temperature the pressure is 101 kPa at sea level and 86.9 kPa at h = 1,000 m. What is the pressure at an altitude of 3500 m?

<u>**Compound Interest**</u>: If A_0 dollars is invested at $r\%$ compounded n times a year, then the amount in the account after t years is given by $A = A_0(1 + r/n)^{nt}$.

EXAMPLE 6: If \$4000 is invested at 8% compounded monthly, how much money is in the account at the end of 6 years?

<u>**Continuous Compound Interest**</u>: If P dollars is invested at $r\%$ compounded continuously, then the amount in the account after t years is given by $A = Pe^{rt}$.

EXAMPLE 7: How much money should be invested now at 6% compounded continuously in order to have $\$30,000$ 18 years from now?

Definition: The rate of cooling of an object is proportional to the temperature difference between the object and the temperature of the object's surroundings. If $y(t)$ is the temperature of the object at time t, then $\frac{dy}{dt} = k(y - T)$, where y is the temperature of the object at time t and T is the room temperature (the temperature of the room in which the object is cooling). The solution of this equation, which gives the temperature of the object at time t, is $y(t) = (y_0 - T)e^{kt} + T$, where y_0 is the initial temperature of the object.

EXAMPLE 8: A thermometer is taken from a room where the temperature is 20°C to the outdoors, where the temperature is 5° C. After one minute, the temperature reads 12°C. Use Newton's Law of Cooling to answer the following questions.

a.) What will the reading of the thermometer be after 2 minutes?

b.) When will the thermemeter read 6°C?