Section 4.5: Exponential Growth and Decay

Definition: If $y(t)$ is the value of a quantity at time t and if the rate of change of y with respect to t is proportional to its size $y(t)$ at any time, [That is $\frac{dy}{dt} = ky$], then the quantity $y(t)$ at time t is given by

$$y(t) = y_0 e^{kt}$$

where y_0 is the initial quantity and k is a constant. Given information, your primary goal is to find k.

EXAMPLE 1: A bacteria culture starts with 4000 bacteria and the population triples every half-hour.

(i) Find an expression for the number of bacteria after t hours.

(ii) Find the number of bacteria after 20 minutes.

EXAMPLE 2: Def: The **half-life** of a substance is the amount of time it takes for half of the substance to disintegrate. Polonium-210 has a half-life of 140 days.

(i) If a sample has a mass of 200 mg, find a formula for the mass that remains after t days.

(ii) When will the mass be reduced to 10 mg?

EXAMPLE 3: After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222? How long will it take the sample to decay to 10% of its original amount?

EXAMPLE 4: A curve passes through the point $(0, 7)$ and has the property that the slope of the curve at every point p is half the y-coordinate of p. Find the equation of the curve.

EXAMPLE 5: A tank contains 1500 liters of brine with a concentration of 0.3 kg of salt per liter. Pure water enters the tank at a rate of 20 liters per minute. The solution is kept mixed and exits the tank at the same rate.

(i) How many kg of salt will remain after half an hour?

(ii) When will the concentration be reduced to 0.2 kg of salt per liter?

Compound Interest: If A_0 dollars is invested at $r\%$ compounded n times a year, then the amount in the account after t years is given by $A = A_0(1 + r/n)^{nt}$.

EXAMPLE 6: If \$4000 is invested at 8% compounded monthly, how much money is in the account at the end of 6 years?

Continuous Compound Interest: If P dollars is invested at $r\%$ compounded continuously, then the amount in the account after t years is given by $A = Pe^{rt}$.

EXAMPLE 7: How much money should be invested now at 6% compounded continuously in order to have \$30,000 18 years from now?

Definition: The rate of cooling of an object is proportional to the temperature difference between the object and the temperature of the object's surroundings. If $y(t)$ is the temperature of the object at time t, then $\frac{dy}{dt} = k(y - T)$, where y is the temperature of the object at time t and T is the room temperature (the temperature of the room in which the object is cooling). The solution of this equation, which gives the temperature of the object at time t, is $y(t) = (y_0 - T)e^{kt} + T$, where y_0 is the initial temperature of the object.

EXAMPLE 8: A thermometer is taken from a room where the temperature is 20°C to the outdoors, where the temperature is 5° C. After one minute, the temperature reads 12°C. Use Newton's Law of Cooling to answer the following questions.

a.) What will the reading of the thermometer be after 2 minutes?

b.) When will the thermemeter read 6°C?