Section 4.8: L'Hospital's Rule

Indeterminate form: If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then we say the limit is in indeterminate form.

L'Hospital's Rule: If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

Example 1: Find the following limits, if they exist. If the limit does not exist, explain why.

(i) $\lim_{x \to 1} \frac{\ln x}{x - 1}$

(ii) $\lim_{x \to 0} \frac{\sin x - x}{x^3}$

(iii) $\lim_{x \to \infty} \frac{(\ln x)^3}{x^2}$

Indeterminate Products: If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = \infty$, then $\lim_{x \to a} f(x)g(x)$ is an indeterminate product. Why do we call the product indeterminate?

Example 2: Find the following limits, if they exist. If the limit does not exist, explain why.

(i) $\lim_{x \to 0^+} x^3 \ln x$

(ii) $\lim_{x \to -\infty} xe^x$

Indeterminate Powers: If $\lim_{x \to a} f(x)^{g(x)}$ is of the form 0^0, ∞^0 or 1^∞, then the limit is an indeterminate power.

Example 3: Find the following limits, if they exist. If the limit does not exist, explain why.

(i) $\lim_{x \to 0^+} (\sin x)^{\tan x}$

(ii) $\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^x$

(iii) $\lim_{x\to\infty} x^{\frac{3}{x}}$

Indeterminate difference: If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then $\lim_{x\to a}(f(x) - g(x))$ is an indeterminate difference.

Example 4: Find $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$