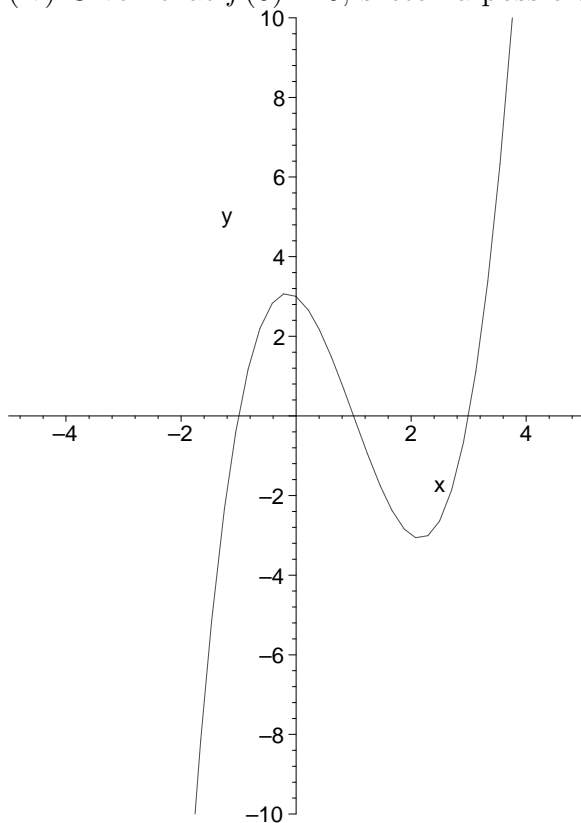


Section 5.1: What does  $f'$  say about  $f$ ?**What does  $f'$  say about  $f$ ?**

- If  $f' > 0$  on an interval  $I$ , then  $f$  is increasing on  $I$ .
- If  $f' < 0$  on an interval  $I$ , then  $f$  is decreasing on  $I$ .
- If  $f'$  goes from positive to negative at  $x = a$ , and  $x = a$  is in the domain of  $f$ , then  $f$  has a local maximum at  $x = a$ .
- If  $f'$  goes from negative to positive at  $x = a$ , and  $x = a$  is in the domain of  $f$ , then  $f$  has a local minimum at  $x = a$ .

*EXAMPLE 1:* Below is the graph of the derivative,  $f'$ , of some function  $f$ . Use it to answer the following questions:

- On what intervals is  $f$  increasing?
- On what intervals is  $f$  decreasing?
- At what  $x$  values does  $f$  have a local maximum or minimum?
- Given that  $f(0) = 0$ , sketch a possible graph of  $f$ .



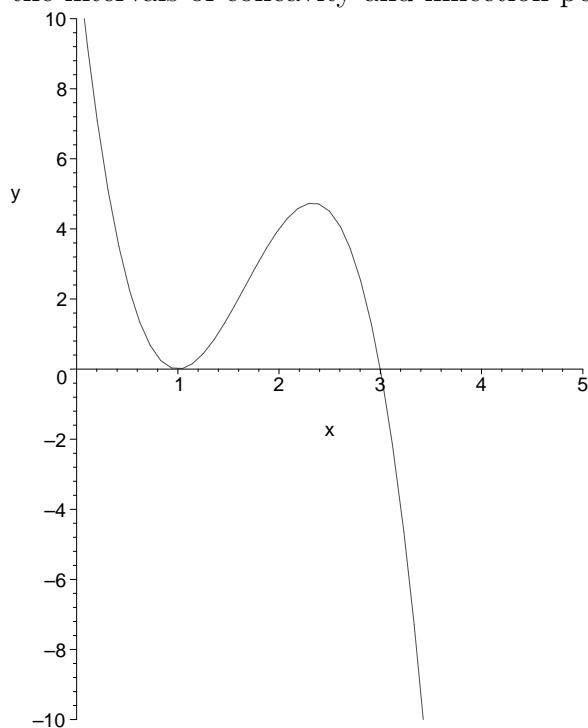
**Definition** If the slopes of a curve become progressively larger as  $x$  increases, then we say  $f$  is **concave upward**. If the slopes of a curve become progressively smaller as  $x$  increases, then we say  $f$  is **concave downward**.

Illustration:

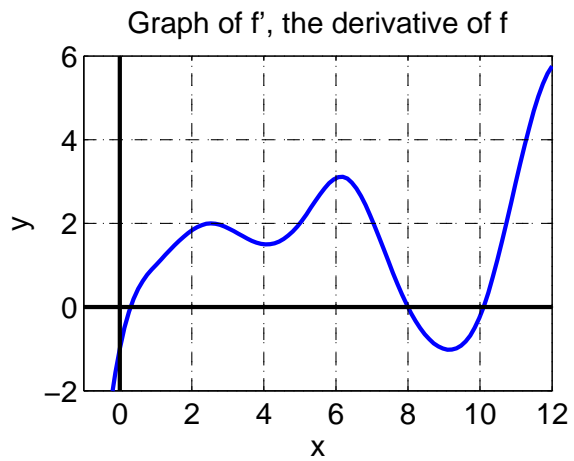
What does  $f''$  say about  $f$ ?

- If  $f'' > 0$  on an interval  $I$ , then  $f'$  is increasing, hence  $f$  is concave up on  $I$ .
- If  $f'' < 0$  on an interval  $I$ , then  $f'$  is decreasing, hence  $f$  is concave down on  $I$ .
- If  $f$  changes concavity at  $x = a$ , and  $x = a$  is in the domain of  $f$ , then  $x = a$  is an inflection point of  $f$ .

*EXAMPLE 2:* Below is the graph of the derivative,  $f'$ , for some function  $f$ . Find the intervals of concavity and inflection point(s).



*EXAMPLE 3:* Below is the graph of the derivative,  $f'$ , of some function  $f$ . Using the fact that  $f(0) = 0$ , sketch a possible graph of  $f$ .



*EXAMPLE 4:* Sketch a graph of  $f$  satisfying the following conditions:

- (i)  $f'(x) > 0$  on the interval  $(-\infty, 1)$  and  $f'(x) < 0$  on the interval  $(1, \infty)$ .
- (ii)  $f''(x) > 0$  on the interval  $(-\infty, -2)$  and  $(2, \infty)$ .
- (iii)  $f''(x) < 0$  on the interval  $(-2, 2)$ .
- (iv)  $\lim_{x \rightarrow -\infty} f(x) = -2$  and  $\lim_{x \rightarrow \infty} f(x) = 0$ .

*EXAMPLE 5:* If  $f'(4) = 0$  and  $f''(4) = 5$ , what can be said about  $f$ ?