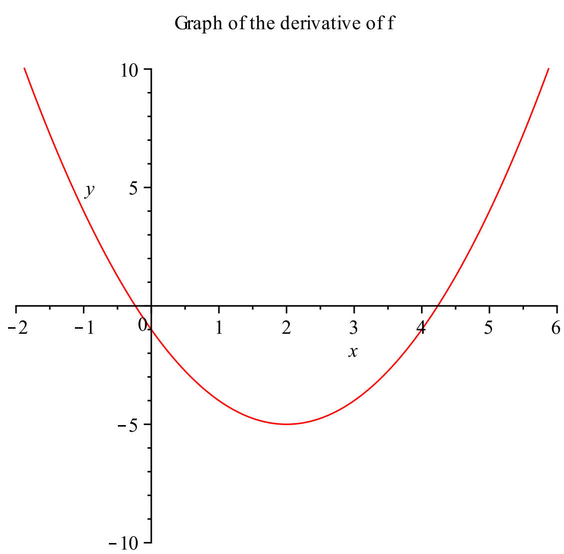
Section 5.1: What does f' say about f?

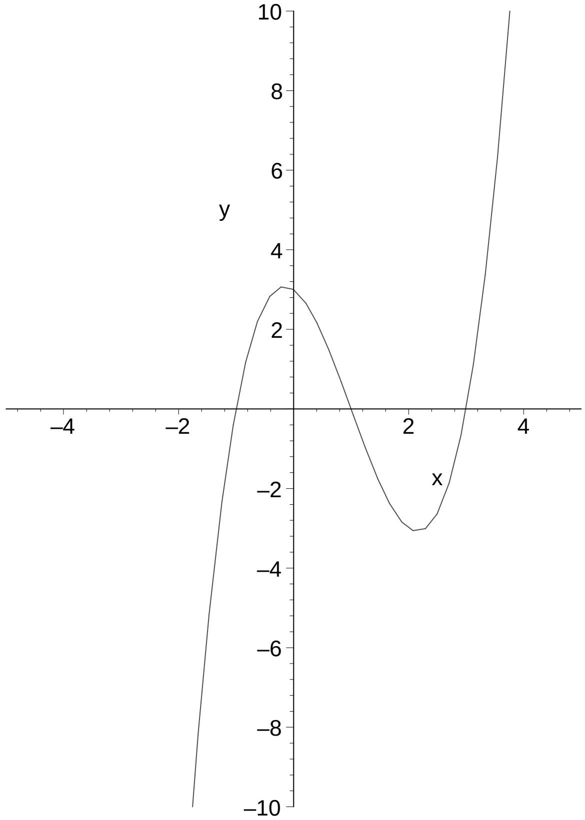
What does f' say about f?

- If $f' > 0$ on an interval I, then f is increasing on I.
- If $f' < 0$ on an interval I, then f is decreasing on I.
- If f' goes from positive to negative at $x = a$, and $x = a$ is in the domain of f, then f has a local maximum at $x = a$.
- If f' goes from negative to positive at $x = a$, and $x = a$ is in the domain of f, then f has a local minimum at $x = a$.

Illustration:



EXAMPLE 1: Below is the graph of the derivative, f', of some function f. Use it to answer the following questions:



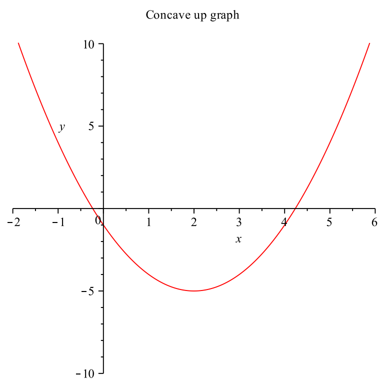
(i) On what intervals is f increasing?

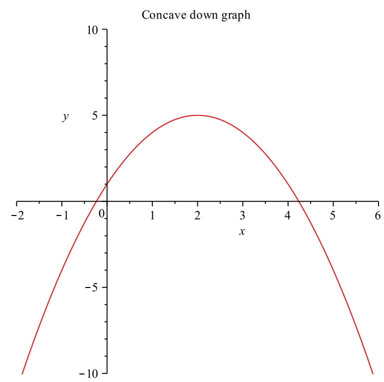
(ii) On what intervals is f decreasing?

(iii) At what x values does f have a local maximum or minimum?

Definition If the slopes of a curve become progressively larger as x increases, then we say f is **concave upward.** If the slopes of a curve become progressively smaller as x increases, then we say f is **concave downward.**

Illustration:



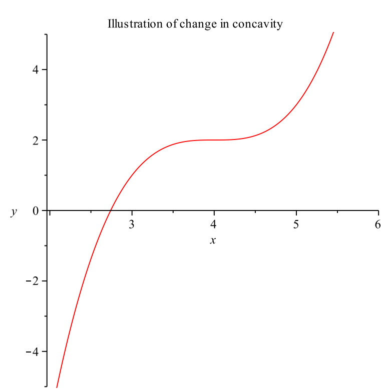


What does f'' say about f?

- If $f'' > 0$ on an interval I, then f' is increasing, hence f is concave up on I.

- If $f'' < 0$ on an interval I, then f' is decreasing, hence f is concave down on I.

- If f changes concavity at $x = a$, and $x = a$ is in the domain of f, then $x = a$ is an inflection point of f.



EXAMPLE 2: If $f'(4) = 0$ and $f''(4) = 5$, what can be said about f?

EXAMPLE 3: If $f'(x) = e^{-x^2}$ what can be said about f?

EXAMPLE 4: Sketch a graph of f satisfying the following conditions:

(i) $f'(x) > 0$ on the interval $(-\infty, 1)$ and $f'(x) < 0$ on the interval $(1, \infty)$.

(ii) $f''(x) > 0$ on the interval $(-\infty, -2)$ and $(2, \infty)$.

(iii) $f''(x) < 0$ on the interval $(-2, 2)$.

(iv) $\lim_{x \to -\infty} f(x) = -2$ and $\lim_{x \to \infty} f(x) = 0$.