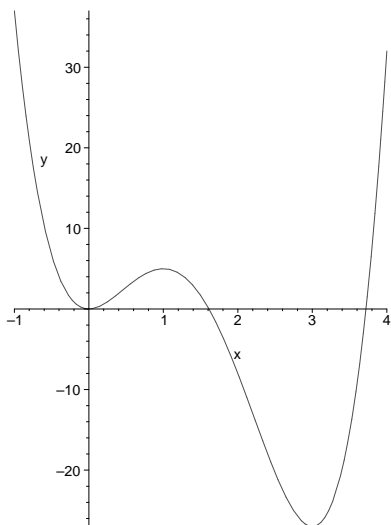


## Section 5.2: Maximum and Minimum Values

**Definition**

(1) A function  $f$  has an **absolute maximum** at  $c$  if  $f(c) \geq f(x)$  for all  $x$  in the domain of  $f$ . A function  $f$  has an **absolute minimum** at  $c$  if  $f(c) \leq f(x)$  for all  $x$  in the domain of  $f$ . In this case, we call  $f(c)$  the **maximum value** or **minimum value**, respectively.

(2) A function  $f$  has a **local maximum** at  $c$  if  $f(c) \geq f(x)$  when  $x$  is near  $c$ . A function  $f$  has a **local minimum** at  $c$  if  $f(c) \leq f(x)$  when  $x$  is near  $c$ .



*EXAMPLE 1:* Find all absolute and local extrema by graphing the function:

(a)  $f(x) = 1 - x^2$ ,  $0 \leq x \leq 1$ .

(b)  $f(x) = 1 - x^2$ ,  $-1 \leq x \leq \frac{1}{2}$ .

$$(c) f(x) = \frac{1}{x}, 0 < x < 1.$$

$$(d) f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 0 \\ 2 - x^2 & \text{if } 0 \leq x \leq 1 \end{cases}$$

**Extreme Value Theorem** If  $f(x)$  is a continuous function on a closed interval  $[a, b]$ , then  $f$  will attain both an absolute maximum and an absolute minimum.

*EXAMPLE 2:* Graph an example of a continuous function on a non closed interval that does not attain an absolute maximum but does attain an absolute minimum.

*EXAMPLE 3:* Graph an example of a discontinuous function on a closed interval that does not attain an absolute minimum but does attain an absolute maximum.

**Definition** We call  $x = c$  a **critical number** of  $f(x)$  if  $x = c$  is in the domain of  $f$  and either  $f'(c) = 0$  or  $f'(c)$  does not exist.

*EXAMPLE 4:* Find all critical values for the following functions:

(a)  $f(x) = 4x^3 - 9x^2 - 12x + 3$

(b)  $f(x) = \sqrt[3]{x}$

(c)  $f(x) = |x^2 - 1|$

(d)  $f(x) = xe^{2x}$

*EXAMPLE 5:* Use the extreme value theorem to find the absolute extrema for  $f(x) = 1 + 27x - x^3$  on the interval  $[0, 4]$ .

*EXAMPLE 6:* Use the extreme value theorem to find the absolute extrema for  $f(x) = x - 2 \cos x$  on the interval  $[0, \pi]$ .