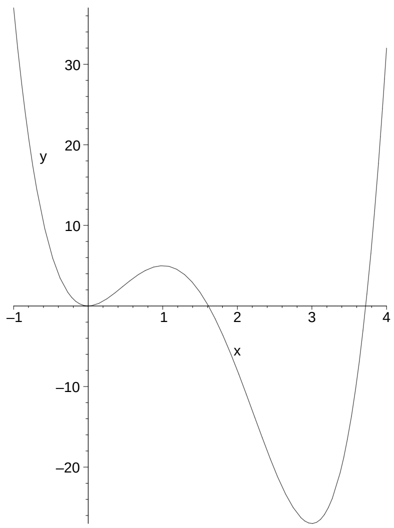
Section 5.2: Maximum and Minimum Values

Definition

(1) A function f has an **absolute maximum** at c if $f(c) \geq f(x)$ for all x in the domain of f. A function f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in the domain of f. In this case, we call $f(c)$ the **maximum value** or **minimum value**, respectively.

(2) A function f has a **local maximum** at c if $f(c) \geq f(x)$ when x is near c. A function f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.



EXAMPLE 1: Find all absolute and local extrema by graphing the function:

(a) $f(x) = 1 - x^2$, $-3 < x \leq 2$.

(b) $f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 0 \\ 2 - x^2 & \text{if } 0 \leq x \leq 1 \end{cases}$

<u>Definition</u> We call $x = c$ a **critical number** of $f(x)$ if $x = c$ is in the domain of f and either $f'(c) = 0$ or $f'(c)$ does not exist.

EXAMPLE 2: Find the domain and critical values for the following functions:

(a) $f(x) = 4x^3 - 9x^2 - 12x + 3$

(b) $f(x) = |x^2 - 1|$

(c) $f(x) = \sqrt[3]{x^2 - 3x}$

(d) $f(x) = xe^{2x}$

(e) $f(x) = x \ln x$

<u>Extreme Value Theorem</u> If $f(x)$ is a continuous function on a closed interval $[a, b]$, then f will attain both an absolute maximum and an absolute minimum.

Graphical illustration:

EXAMPLE 3: Use the extreme value theorom to find the absolute extrema for $f(x) = 1 + 27x - x^3$ on the interval $[0, 4]$.

EXAMPLE 4: Use the extreme value theorom to find the absolute extrema for $f(x) = x - 2\cos x$ on the interval $[0, \pi]$.