Section 5.7: Antiderivatives

Definition We call $F(x)$ an antiderivative of $f(x)$ if $F'(x) = f(x)$.
For example, x^2 is an antiderivative of $2x$ because $\frac{d}{dx}(x^2) = 2x$.

Definition If F is an antiderivative of f, then the most general antiderivative of f is $F(x) + C$, where C is any arbitrary constant.

Table of Antiderivatives:

Function	Antiderivative
k	$kx + c$
x^n, if $n \neq -1$	$\frac{x^{n+1}}{n+1} + c$
x^{-1}	$\ln\|x\| + c$
e^x	$e^x + c$
a^x	$\frac{a^x}{\ln a} + c$
$\cos x$	$\sin x + c$
$\sin x$	$-\cos x + c$
$\sec x \tan x$	$\sec x + c$
$\sec^2 x$	$\tan x + c$
$\csc^2 x$	$-\cot x + c$
$\csc x \cot x$	$-\csc x + c$
$\frac{1}{x^2+1}$	$\arctan x + c$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + c$

EXAMPLE 1: Find the most general antiderivative.

(i) $f(x) = x^3 - 4x^2 + e$

(ii) $f(x) = \sqrt[3]{x^2} - \sqrt{x^3}$

(iii) $f(x) = \dfrac{x + 5x^2 - 1}{2x^3}$

(iv) $f(x) = e^x + \dfrac{4}{\sqrt{1 - x^2}} + 5(1 - x^2)^{-1/2}$

EXAMPLE 2: Given the graph of f passes through the point $(1, 6)$ and the slope of its tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

EXAMPLE 3: A particle is moving according to acceleration $a(t) = 3t + 8$. Find the position, $s(t)$, of the object at time t if we know $s(0) = 1$ and $v(0) = -2$.

EXAMPLE 4: Suppose the acceleration of an object at time t is given by $\mathbf{a}(t) = (\cos t\mathbf{i}) - 3\mathbf{j}$. Find the position vector function, $\mathbf{r}(t)$, if it is known that $\mathbf{v}(2) = \mathbf{i} - \mathbf{j}$ and $\mathbf{r}(0) = \mathbf{0}$.

EXAMPLE 5: If $f''(x) = x^2$, $f(1) = 2$ and $f(2) = 3$, find $f(x)$.

EXAMPLE 6: A car braked with a constant deceleration of 40 feet per second squared, producing skid marks measuring 160 feet before coming to a stop. How fast was the car traveling when the brakes were first applied?