Section 6.1: Sigma Notation

Definition If m and n are a non negative integers, with $m < n$, then $\sum_{i=m}^{n} a_i = a_m + a_2 + ... + a_n$. We call i the index of the sum, m the lower limit of the sum, and n the upper limit of the sum.

EXAMPLE 1: Evaluate $\sum_{i=1}^{6} \frac{1}{i+1}$.

EXAMPLE 2: Expand $\sum_{k=5}^{8} x^k$.

EXAMPLE 3: Write $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36}$ in sigma notation.

Theorem

- $\sum_{i=m}^{n} ca_i = c \sum_{i=m}^{n} a_i$.
- $\sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i$.
- $\sum_{i=1}^{n} (1) = n$
- $\sum_{i=1}^{n} (c) = cn$
- $\sum_{i=1}^{n} (i) = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} (i^2) = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} (i^3) = \left(\frac{n(n+1)}{2}\right)^2$

EXAMPLE 4: Evaluate the following:

(i) $\sum_{i=1}^{50}(4)$

(ii) $\sum_{i=3}^{25}(7)$

(iii) $\sum_{i=1}^{100} 6i$

(iv) $\sum_{i=3}^{99}\left(\frac{1}{i} - \frac{1}{i+1}\right)$

(v) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left[\left(\frac{i}{n} \right)^3 + 1 \right]$

EXAMPLE 5: If $\sum_{i=1}^{4} f(i) = 3$ and $\sum_{i=1}^{4} g(i) = 13$, find $\sum_{i=1}^{4} (2f(i) + g(i))$