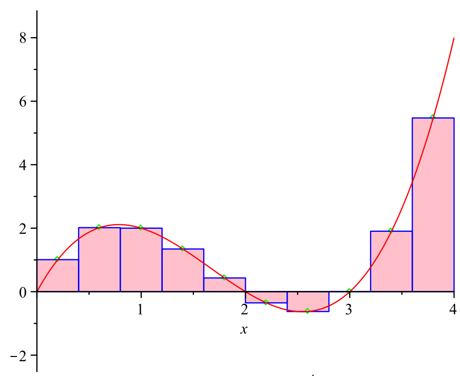
# Section 6.4: The Fundamental Theorem of Calculus

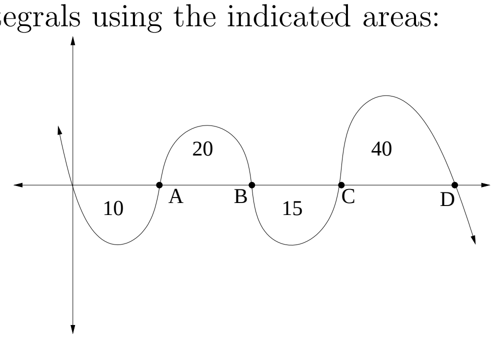
Recall $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where $\Delta x_i = \dfrac{b-a}{n}$ and $x_i^*$ is any point on the $i$th subinterval. In the event $f(x)$ is positive on the interval $[a,b]$, then the definite integral is the same as the area bounded by $f(x)$, the $x$-axis, $x=a$ and $x=b$. If $f(x)$ is not always positive on the interval $[a,b]$, then the definite integral is the net area.

For the graph provided below, $\int_0^4 f(x)\,dx \approx \sum_{i=1}^{10} f(x_i^*)\Delta x_i$, where $x_i$ is the midpoint of the $ith$ subinterval.




A midpoint Riemann sum approximation of $\int_0^4 f(x)\,dx$, where $f(x) = x(x-2)(x-3)$ and the partition is uniform. The approximate value of the integral is 5.280000000. Number of partitions used: 10.

*EXAMPLE 1:* Refer to the graph provided below to compute the following definite integrals using the indicated areas:




$\int_0^A f(x)\,dx$

$\int_0^C f(x)\,dx$

$\int_A^B f(x)\,dx$

$\int_0^D f(x)\,dx$

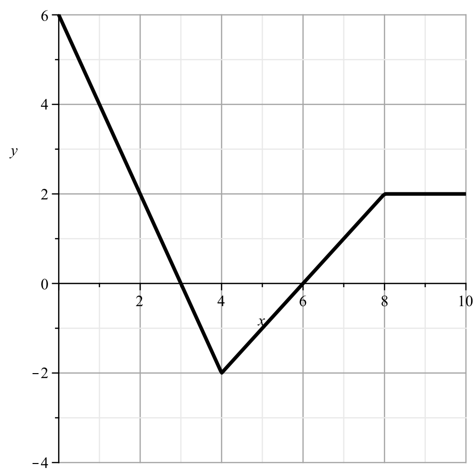
$\int_C^A f(x)\,dx$

$\int_C^C f(x)\,dx$

**The Fundamental Theorem of Calculus**

- Part I: If $f$ is continuous on $[a, b]$, then the function $g$ defined by $g(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$ and differentiable on $(a, b)$ and $g'(x) = f(x)$.
- Part II: If $f$ is continuous on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F$ is an antiderivative of $f$.

*EXAMPLE 2:* If $g(x) = \int_0^x f(t)\,dt$, where the graph of $f(t)$ is given below, where $0 \leq x \leq 10$, evaluate $g(0)$, $g(3)$, $g(6)$ and $g(10)$. Where is $g(x)$ increasing? What is the maximum value of $g(x)$?



*EXAMPLE 3:* Find the derivative of:

(a) $g(x) = \int_{-1}^{x} \sqrt{t^3 + 1}\, dt$

(b) $g(x) = \int_{1}^{\sqrt{x}} \frac{t^2}{t^2 + 1}\, dt$

(c) $g(x) = \int_{x^2}^{\sin x} \frac{\cos t}{t}\, dt$

*EXAMPLE 4:*

(a) Evaluate $\int_1^2 \frac{1}{x^2}\,dx$

(b) Evaluate $\int_{-1}^0 (5x^2 - 4x + 3)\,dx$

(c) Evaluate $\int_{\ln 3}^{\ln 6} 8e^t\,dt$

(d) Evaluate $\int_1^2 \frac{x^6 - x^2}{x^7}\, dx$

(e) Evaluate $\int_0^2 (w^3 - 1)^2\, dw$

(f) Evaluate $\int_{-2}^0 |x^2 - 1|\, dx$

(g) Find $\int_0^{\pi/2} f(x)\,dx$ where $f(x) = \begin{cases} \cos x & \text{if } 0 \le x < \frac{\pi}{4} \\ \sin x & \text{if } \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$

*EXAMPLE 5:* Suppose an object is moving according to velocity $v(t) = 3t - 5$, $0 \le t \le 3$. Find the displacement and distance traveled during the first 3 seconds.