

## Section 6.5: Integration by Substitution

**The Substitution Rule:** If  $u = g(x)$  is a differentiable function, then

$$\int f(g(x))g'(x) dx = \int f(u) du$$

*EXAMPLE 1:* Integrals of the form  $\int (f(x))^n f'(x) dx$

a.)  $\int x(x^2 + 3)^{10} dx$

b.)  $\int \frac{2t^2}{\sqrt{t^3 - 1}} dt$

c.)  $\int_0^1 \frac{x}{(x+1)^2} dx$

*EXAMPLE 2:* Integrals of the form  $\int e^{f(x)} f'(x) dx$

a.)  $\int_{-1}^2 x e^{x^2} dx$

*EXAMPLE 3:* Integrals of the form  $\int \frac{f'(x)}{f(x)} dx$

a.)  $\int_0^1 \frac{x}{x^2 + 1} dx$

b.)  $\int \frac{ax + b}{ax^2 + 2bx + c} dx$

c.)  $\int \tan(x) dx$

$$\text{d.) } \int_{e^2}^{e^3} \frac{1}{x \ln x} dx$$

*EXAMPLE 4:* Integrals of the form  $\int \cos(f(x))f'(x) dx$ ,  $\int \sin(f(x))f'(x) dx$ ,  $\int \sec^2(f(x))f'(x) dx$ ,  
 $\int \sec(f(x)) \tan(f(x))f'(x) dx$ ,  $\int \csc^2(f(x))f'(x) dx$ ,  $\int \csc(f(x)) \cot(f(x))f'(x) dx$

$$\text{a.) } \int x \cos(2 - x^2) dx$$

$$\text{b.) } \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$$