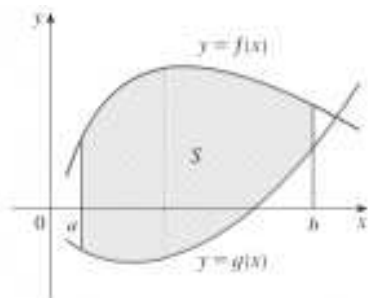


Section 7.1: Area

Area

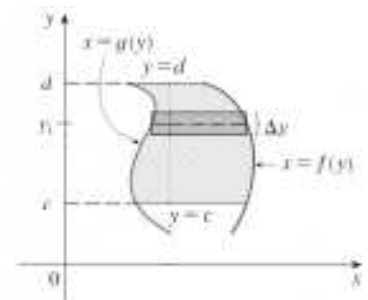
- The area bounded by the curves $y = f(x)$, $y = g(x)$ and the lines $x = a$ and $x = b$, where $f(x) \geq g(x)$ for all x in the interval $[a, b]$ is

$$S = \int_a^b (f(x) - g(x)) \, dx$$



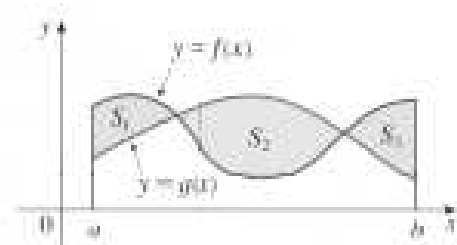
- The area bounded by the curves $x = f(y)$, $x = g(y)$ and the lines $y = c$ and $y = d$, where $f(y) \geq g(y)$ for all y in the interval $[c, d]$ is

$$A = \int_c^d (f(y) - g(y)) \, dy$$



- If we are asked to find the area bounded by the curves $y = f(x)$, $y = g(x)$ where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x , we must split the integral at each intersection point.

$$S = S_1 + S_2 + S_3$$



EXAMPLE 1: Find the area bounded by $y = \sin x$, $y = 0$, $x = 0$, $x = \frac{\pi}{4}$.

EXAMPLE 2: Find the area bounded by $y = \cos x$, $y = 0$, $x = 0$, $x = \frac{2\pi}{3}$.

EXAMPLE 3: Find the area bounded by $y = \frac{1}{x}$, $y = -1$, $x = 1$, $x = 3$.

EXAMPLE 4: Find the area bounded by $y = 4x^2$, $y = x^2 + 3$.

EXAMPLE 5: Find the area bounded by $y^2 = x$, $x - 2y = 3$.

EXAMPLE 6: Find the area bounded by $y = x^2$, $y = \frac{2}{x^2 + 1}$.

EXAMPLE 7: Find the area bounded by $y = \sin x$, $y = \cos x$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$.

EXAMPLE 8: Find the area bounded by $y = \cos x$, $y = \sin 2x$, $x = 0$, $x = \frac{\pi}{2}$

EXAMPLE 9: Find the area bounded by $y = |x^2 - 1|$, $y = 0$, $x = 0$, $x = 2$.

EXAMPLE 10: Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$ and the x -axis.