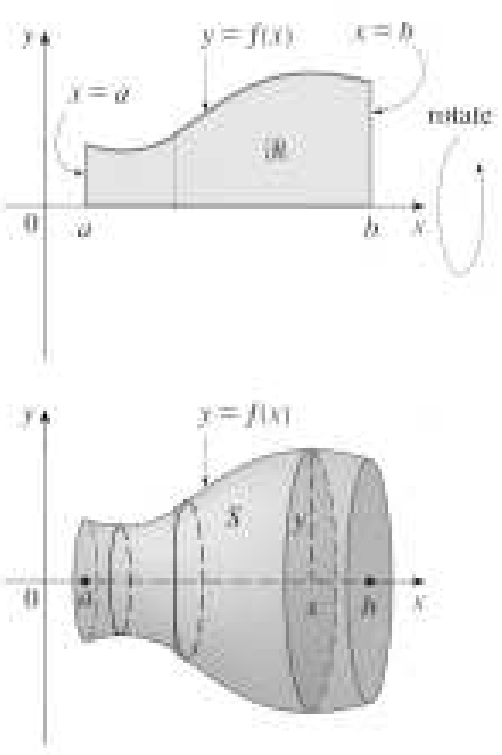
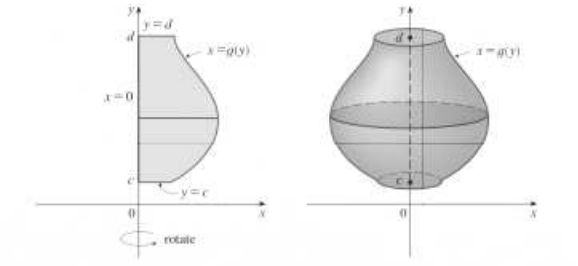
Section 7.2: Volume

1. Disk Method: Use when the cross-section of the solid is in the shape of a disk

   - Revolution around the x-axis: $V = \int_a^b \pi \left(f(x)\right)^2 \, dx$



   - Revolution around the y-axis: $V = \int_c^d \pi \left(g(y)\right)^2 \, dy$



EXAMPLE 1: Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x-1}$, $x = 2$, $x = 5$, $y = 0$ about the x axis.

EXAMPLE 2: Find the volume of the solid obtained by rotating the region bounded by $y = \tan x$, $x = 0$, $x = \dfrac{\pi}{4}$, $y = 0$ about the x axis.

EXAMPLE 3: Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 27$, $x = 0$ about the y axis.

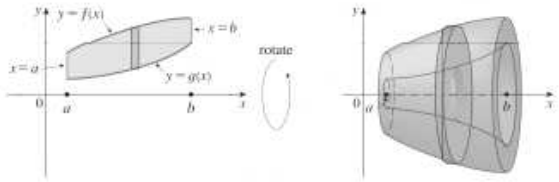
EXAMPLE 4: Find the volume of the solid obtained by rotating the region bounded by $y = \ln x$, $x = 0$, $y = 0$, $y = 5$, about the y axis.

- Revolution around lines:

EXAMPLE 5: Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$.

EXAMPLE 6: Find the volume of the solid obtained by rotating the region bounded by $y = x^2 + 1$, $x = 0$, $y = 10$ about the line $y = 10$.

2. Washer Method: Use when the cross-section of the solid is in the shape of a washer
$V = \int_a^b \pi \left((f(x))^2 - (g(x))^2 \right) \, dx$



EXAMPLE 7: Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 2x$, about the x axis.

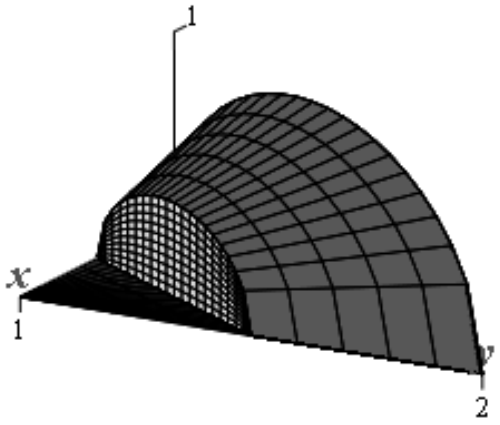
EXAMPLE 8: Find the volume of the solid obtained by rotating the region bounded by $y = 2x$, $y = 0$, $x = 1$, $x = 2$, about the y axis.

EXAMPLE 9: Find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = x^2$, about the line $y = 2$.

EXAMPLE 10: Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 4$, $y = 0$, about the line $x = 5$.

3. The Method of Slicing

EXAMPLE 11: Find the volume of the solid S whose base is the triangular region with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$. The cross sections of S perpendicular to the x-axis are semi-circles.



EXAMPLE 12: Find the volume of the solid S whose base is the region bounded by the parabola $y = x^2$ and $y = 1$. The cross sections of S perpendicular to the y-axis are equilateral triangles.

EXAMPLE 13: Find the volume of the solid S whose base is the ellipse $x^2 + \frac{y^2}{4} = 1$. The cross sections of S perpendicular to the y-axis are squares.

EXAMPLE 14: Find the volume of the cap of a sphere with radius 4 and height 1.