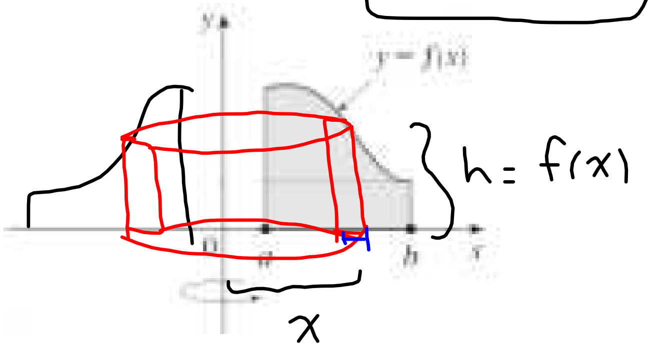
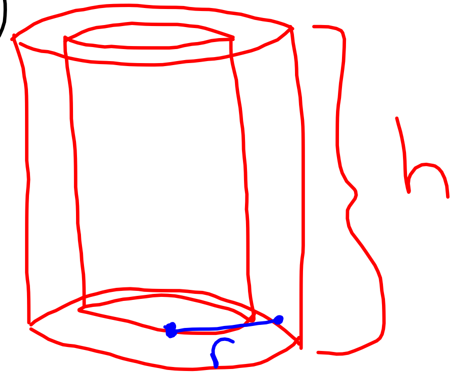
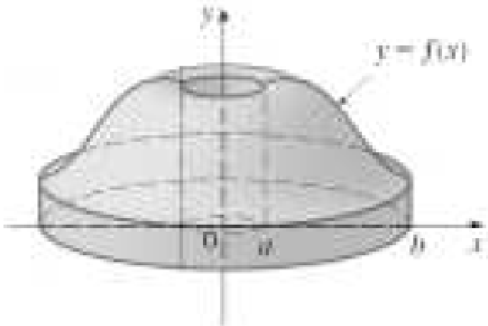
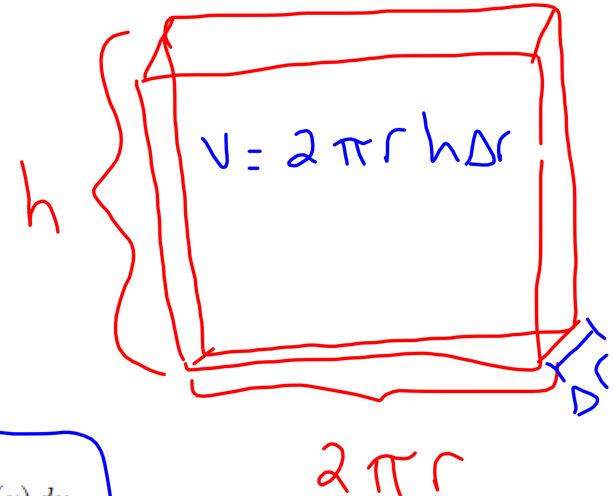
The Method of Cylindrical Shells:

Revolution around the y-axis: $\boxed{V = \int_a^b 2\pi x f(x)\, dx}$

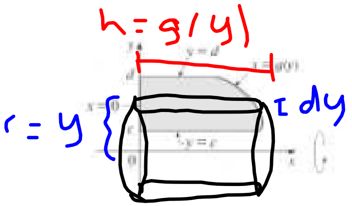


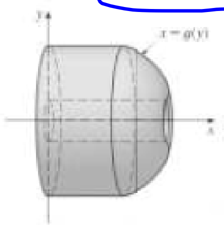




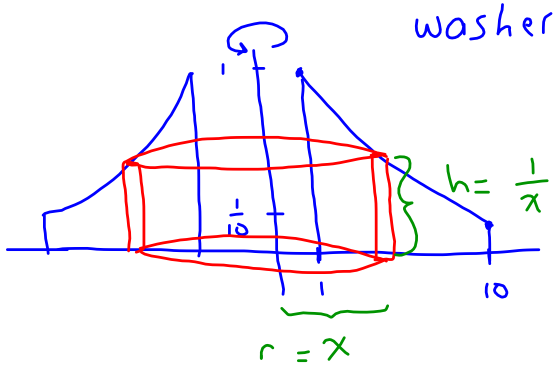


Revolution around the x-axis: $\boxed{V = \int_c^d 2\pi y g(y)\, dy}$





1. Find the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 10$ about the y axis.



washers: $\pi \int_0^{\frac{1}{10}} (R^2 - r^2)\, dy +$

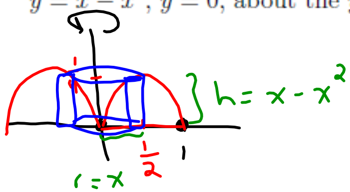
$\pi \int_{\frac{1}{10}}^{1} (R^2 - r^2)\, dy$

$\vdots$

shells:

$$V = \int_1^{10} 2\pi \underbrace{(x)}_{r} \underbrace{\left(\frac{1}{x}\right)}_{h} dx = \int_1^{10} 2\pi\, dx = 18\pi$$

2. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$, $y = 0$, about the y axis.



$y = x - x^2 = x(1-x)$ x-int: $x = 0$, $x = 1$

vertex: $y' = 0$

$1 - 2x = 0$

$x = \frac{1}{2}$, $y = \frac{1}{4}$

$$V = \int_0^1 2\pi (x)(x - x^2)\, dx$$

$$= \int_0^1 2\pi (x^2 - x^3)\, dx = 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1$$

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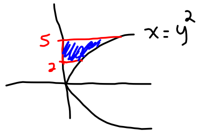
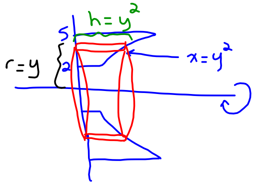
3. Find the volume of the solid obtained by rotating the region bounded by $y^2 = x$, $x = 0$, $y = 2$, $y = 5$ about the x axis.

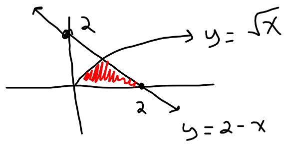


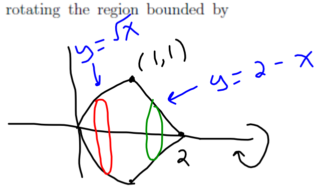


$$V = \int_2^5 2\pi \underbrace{y}_{r}\underbrace{y^2}_{h}\,dy = \int_2^5 2\pi y^3\,dy = 2\pi \frac{y^4}{4}\Big|_2^5$$

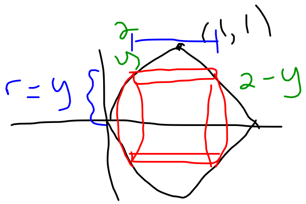
4. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, $x + y = 2$, about the x axis.

$y = \sqrt{x}$ $\quad y = 2 - x$





① $V = \int_0^1 \pi(\sqrt{x})^2\,dx + \int_1^2 \pi(2-x)^2\,dx$



① $y = \sqrt{x}$

$x = y^2$

② $y = 2 - x$

$x = 2 - y$

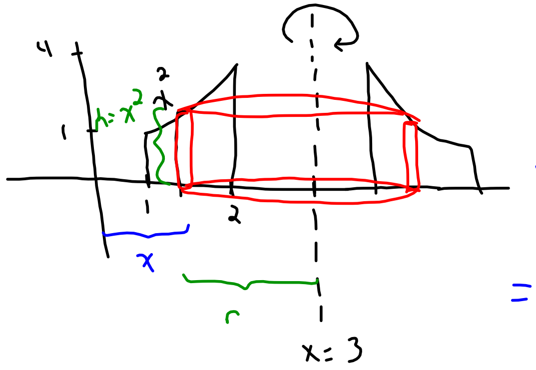
$r = y$

$h = 2 - y - y^2$

$$V = \int_0^1 2\pi y(2 - y - y^2)\,dy$$

Rotation around lines

5. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 0$, $x = 1$, $x = 2$ about the line $x = 3$.



$r = 3 - x$

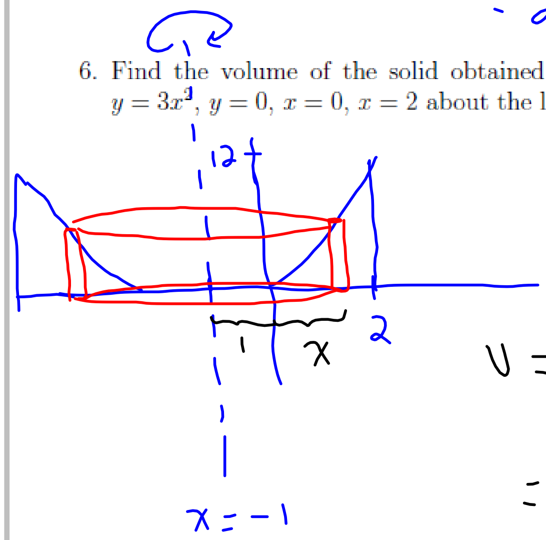
$h = x^2$

$$V = \int_1^2 2\pi (3-x)(x^2)\,dx$$

$$= 2\pi \int_1^2 (3x^2 - x^3)\,dx$$

$$= 2\pi \left(x^3 - \frac{x^4}{4} \right)\Big|_1^2 = \_\_\_$$

6. Find the volume of the solid obtained by rotating the region bounded by $y = 3x^2$, $y = 0$, $x = 0$, $x = 2$ about the line $x = -1$.



$r = x + 1$

$h = 3x^2$

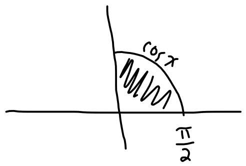
$$V = \int_0^2 2\pi (x+1)(3x^2)\,dx$$

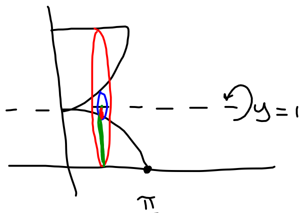
$$= 2\pi \int_0^2 (3x^3 + 3x^2)\,dx$$

$$= 2\pi \left(\frac{3x^4}{4} + x^3 \right)\Big|_0^2$$

$$= \_\_\_$$

7. Find the volume of the solid obtained by rotating the region bounded by $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$, $y = 0$, about the line $y = 1$. Do not evaluate the integral.



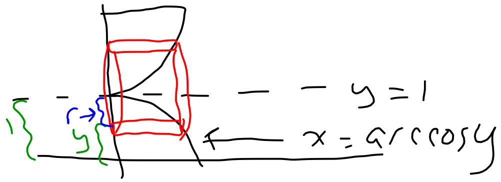


washers:

R = outer radius = 1

r = inner radius = $1-\cos x$

$$V = \int_0^{\frac{\pi}{2}} \pi \left[1^2 - (1-\cos x)^2 \right] dx$$



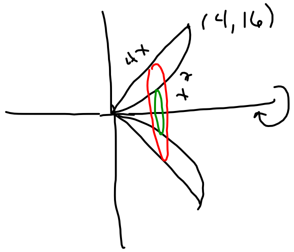
$h = \arccos y$

$r = 1 - y$

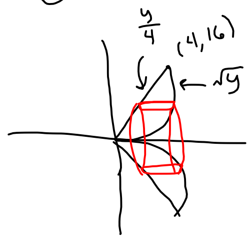
shells: $$V = \int_0^1 2\pi (1-y)(\arccos y)\, dy$$

Choosing a method Often times, more than one method can be used to find the volume.

8. Consider the region bounded by $y = x^2$ and $y = 4x$. Rotate this region about the x axis. Set up the integral to find the volume using both washers and shells. Do not evaluate either integral.



$$V = \int_0^4 \pi \left[(4x)^2 - (x^2)^2 \right] dx$$



$$V = \int_0^{16} 2\pi (y)\left(\sqrt{y} - \frac{y}{4}\right) dy$$

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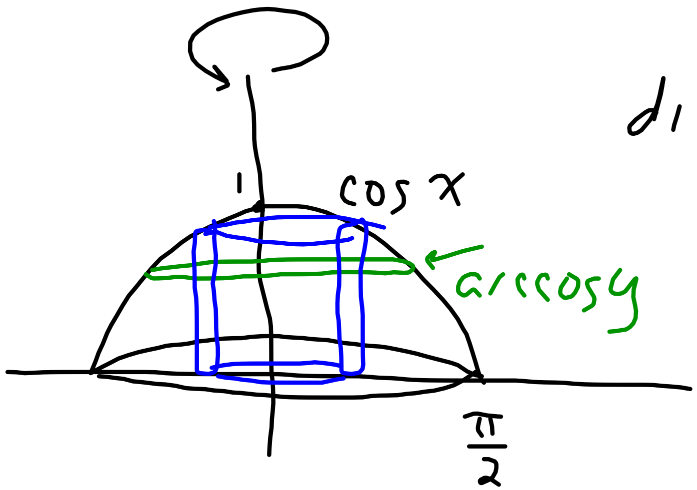
9. Consider the region bounded by $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$, $y = 0$. Rotate this region about the y axis. Set up the integral to find the volume using both disks and shells. Do not evaluate either integral.



disk: $V = \int_0^1 \pi (\arccos y)^2 \, dy$

shell: $V = \int_0^{\frac{\pi}{2}} 2\pi x \cos x \, dx$